International Journal of Information Technology and Knowledge Management
July-December 2008, Volume 1, No. 2, pp. 439-448

PREDICTION OF RELIABILITY OF MULTISTAGE INTERCONNECTION NETWORKS BY MULTI-DECOMPOSITION METHOD

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Abstract

This paper introduces a new method based on multi-decomposition for predicting the exact reliability of fault-tolerant multistage networks. The method is well supported by an efficient algorithm which runs polynomially. The proposed method is found to be simple, general and efficient and thus is as such applicable to all the fault-tolerant multistage interconnection networks. The proposed method is compared with existing technique to evaluate the reliability. The results show this method provides a greater accurate probability when applied on fault-tolerant multistage interconnection networks. Reliability of some important MINs are evaluated and compared under the same platform.

Keywords: Probabilistic graph, Reliability, Fault-tolerant, Interconnection network

1. Introduction

Almost all of the parallel computer interconnection networks can be broadly classified into two groups: static networks and dynamic networks. Static networks are formed of point-to-point direct connections which will not change during program execution. On the other hand, dynamic networks are implemented by switched channels, which are dynamically configured to match the communication demand in use programs. There are three well known types of dynamic interconnection network: Crossbar networks, bus networks and multistage interconnection networks. Out of these networks multistage interconnection networks have attracted great interest recently[2], [5], [15], [16]. Multistage Interconnection Network (MIN) [24] is a low cost network, which interconnects N inputs with N outputs and has log_2N switching stages. Each stage consists of 2 × 2 switching elements with or without loop dependent upon the regular or irregular class of network. Some of the important examples of fault tolerant multistage interconnection networks are Extra stage cube [3], New four Tree Multistage Interconnection Network (NFT) [24], Extra stage shuffle exchange network [23], Multi path Chained Baseline Network (MPN) [1], etc. The main advantages associated with these networks are high bandwidth, low diameter, constant degree switches for which they have been used for various commercial machines including super computers.

Improved performance and increased reliability are the two distinct advantages attributed to interconnection topology. With the increase in size and complexity of the parallel interconnection systems, their reliability becomes extremely important. There are many reliability measures of interest, out of which the node-pair (two terminal) reliability is an important performance measure in parallel computer interconnection systems. Two terminal reliability addresses the probability that a given source-destination pair has at least one fault free path between them. In this context, Blake and Trivedi [7] have developed closed form reliability expressions for two selected multistage interconnection networks (MINs). Subsequently, in a later paper, they have proposed a 2-level hierarchical model in which each sub-system is modeled as a Markov chain and the system reliability of a MIN is modeled as a series system of 'Markov' components. Their methods are good enough for MINs of smaller sizes. Verma and Raghavendra [21] considered three redundant–path networks namely Generalized INDRA, Merged Delta and Augmented C-network and derived analytical expressions for them. Colbourn *et al.* [10] proposed an efficient method to compute the source-to-terminal reliability of MINs. However, their method computes only the bounds on reliability, and does not provide the exact solution. Kim *et al.* [8] proposed an analytical model for computing the reliability of hypercube multicomputers. Unfortunately, their model does not generate exact results of reliability and the same method is not applicable for other parallel computer interconnection systems. Tripathy *et al.* [20] predicts the reliability of multistage interconnection networks with multi state elements. Gunwan [23] proposed an analytical technique to find the reliability of extra stage shuffle exchange networks. For gamma networks the reliability has been estimated by using the redundant paths [25]. Reviewing the literature reveals that some other important methods that find the reliability or its related measures [4], [6], [9], [12], [17], [18].

However, the methods discussed so far are not as such applicable for large MINs. So, some approximation techniques must be use for the bounds on reliability [13], [14], [25]. But there is always a demand in finding the exact reliability as it is an important measure of interconnection network. This motivates our study to propose a simple, general and efficient method based on multi-decomposition of networks for predicting the exact value of reliability of MINs.

A new method for evaluating the reliability of MINs is presented in section 2. Section 3 of this paper provides a comparison of this proposed method with a recent method. The reliability of some important fault-tolerant MINs of interest are evaluated and compared. Section 4 concludes the paper.

2. PROPOSED METHOD FOR EVALUATING THE RELIABILITY OF MINS

2.1. Brief Description about the Multi-decomposition Process

In Reliability Logic Graphs (RLG), MINs are represented as directed graphs. The inter change boxes or switching elements (SEs) can be set to one of the four legitimate states: (i) Straight (ii) Lower broadcast (iii) Exchange (iv) Upper broadcast. We consider the internal connectivity of the SEs and retain the complete permutation capability of the MINs. The parallel and cross connection of a switch is determined by the logic level applied at its control line.

In the proposed graph model for MINs, each node represents a link and edges represent switches. Here, only two edges of the graph are used for each state of SE and the other two are utilized in the complementary operating mode. The I/O stages of the ESC networks use SEs with multiplexers and demultiplexers. Enabling and disabling in stages n and O is accomplished with a demultimplexer (DEMUX) at each SE input and a multiplexer (MUX) at its output. The DEMUX and MUX of the ESC have been represented by parallel dotted edges from X_1 to Y_1 and from X_2 to Y_2 respectively in the graphs, as they serve as by-pass to SEs. The family of permutations that could be passed in a conflict-free manner varies from one MIN to other. The full connectivity requires that the outgoing edges in a graph model are always directed to new nodes so that a connection between any inputs to any one of the outputs could be established.

Multiple Decomposition of the RLG

For the purpose of reliability evaluation, a MIN is modeled as a multistage directed graph, denoted by $G\{V, E\}$. For $h = (n+1)$, $V = V_1 U V_2 U \ldots V_h$ is the disjoint union of h sets of vertices, each set being a stage of N vertices. Similarly, $E = E_1 \, U \, E_2 \, U \ldots E_{h-1}$, represents the disjoint union of (h–1) sets of edges, each edge connecting the vertex V_{g+1}. Without loss of generality, we will use the terms vertex and node interchangeably throughout this chapter.

The multistage graph $G\{V, E\}$ having n stages is decomposed into n sub-graphs by taking K^1 minimal cuts through the common nodes between two consecutive stages (Figs. 3.16b, 3.17b, 3.18b). The first sub graph $A(1)$ contains all the source nodes V_s for $s = 1,2, \ldots, N$ and nodes which are common to 1st sub graph and the 2nd sub graph. The nth sub graph $A(n)$ contains all the destination nodes V_t, for $t = 1, 2, \ldots N$, and the nodes common to both the sub graph $A(n)$ and $A(n$–1). All the intermediate sub graphs are designated as $A\ (j + 1)$.

The process is illustrated by taking Extra stage shuffle exchange network as example (Refer Fig. 1(a), (b).

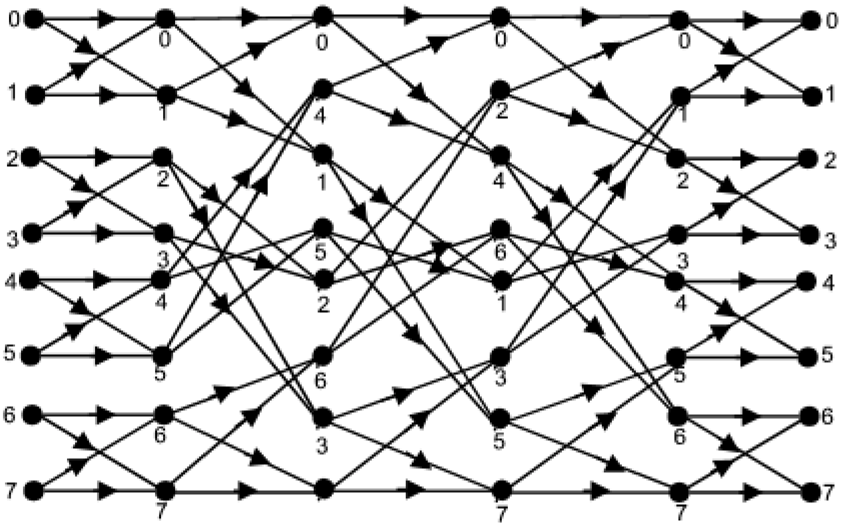

Figure 1(a): Reliability Logic Graph of Extra Stage Shuffle Exchange Network (8 × 8)

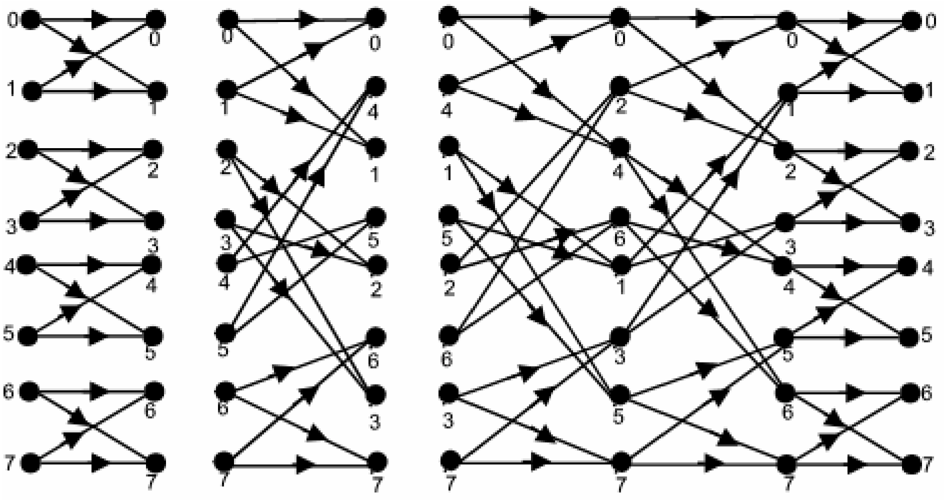

Figure 1(b): Multiple Decomposition of ESEN in Fig. 1(a)

2.3. Algorithm

Multiple_decomposition

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Decompose the graph $G$ into two sub graphs $G_1$ and $G_2$ through a minimal cut-set $C_1$
    if $N_1 > N_2$
        Then decompose $G_1$ into two sub graphs $G_{1,1}$ and $G_{1,2}$ through a minimal cut-set $C_2$
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else
        Decompose $G_2$ into two sub graphs $G_{2,1}$ and $G_{2,2}$ through a minimal cut-set $C_3$
Enumerate all the paths ($P_{1,i,j}$) of sub graph $G_1$ from source node $s$ to $n_i \in C_1$, $i \in C_1$, $j \geq 1$;
for $i = 1$ to $|C_1|$
        $p_{1,i}$ = cardinality of $P_{1,i,j}$;
$X = \phi$;
for $i = 1$ to $|C_1|$
for $j = 1$ to $p_{1,i}$
        $X_i = X_i \cup P_{1,i,j}$;
Enumerate all the paths ($P_{2,i,j}$) of sub graph $G_2$ from source node $n_i$ to $n_j$, $n_i \in C_1$ & $n_j \in C_2$, $i \in C_2$, $j \geq 1$;
for $i = 1$ to $|C_2|$
        $p_{2,i}$ = cardinality of $P_{2,i,j;}$
$Y = \phi$;
for $i$= 1 to $|C_2|$
for $j = 1$ to $p_{2,i}$
        $Y_i = Y_i \cup P_{2,i,j}$;
Enumerate all the paths ($P_{3,i,j}$) of sub graph $G_3$ from source node $n_j$ to $n_i$, $n_j \in N_{C_2}$, $i \in C_3$, $j \geq 1$;
$p_3$ = cardinality of $P_{3,i,j}$;
Z = $\phi$;
for $i = 1$ to $|\mathrm{C}_1|$
for $j = 1$ to $p_{2,i}$
        $Z_i = Z_i P_{3,i,j}$;
                $S_1 = (X_1)_{\text{dis}} (Y_1)_{\text{dis}} \cup ((\bar{X}_1 X_2)_{\text{dis}} (Y_2)_{\text{dis}} \cup (X_1 X_2)_{\text{dis}} (\bar{Y}_1 Y_2)_{\text{dis}})$
                     $\cup ((\bar{X}_1 \bar{X}_2 Xi_3)_{\text{dis}} (Y_3)_{\text{dis}} \cup (\bar{X}_1 X_2 X_3)_{\text{dis}} (\bar{Y}_2 Y_3)_{\text{dis}}$
                     $\cup (X_1 \bar{X}_2 X_3)_{\text{dis}} (\bar{Y}_1 Y_3)_{\text{dis}} \cup (X_1 X_2 X_3)_{\text{dis}} (\bar{Y}_2 \bar{Y}_2 Y_3)_{\text{dis}} \cdots ;$
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/* $(X)_{\text{dis}}$ is similar to calling of the function Dis(X)*/
    $S_2 = (\bar{Y}_1\, Z_1)_{\text{dis}} \cap (\bar{Y}_2 Z_2)_{\text{dis}} \cap (\bar{Y}_3\, Z_3)_{\text{dis}} \cdots ;$
    $S = S_1 \cup S_2;$
    $TR = S_{\{W_i, \bar{W}_i, \cup, \cap\} \rightarrow \{p_i, q_i, +, .\}}$
/*  All indicator variables by their probabilities, the logical sum and product operators
    by their arithmetic counterparts.*/
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Complexity of the Proposed Algorithm

Theorem 1: The proposed algorithm is general and has a worst case running time of N^3.

Proof: As per the definition of MINs, almost all MINs can be converted into their equivalent probabilistic graphs. The proposed algorithm inputs a graph and finds its reliability. Hence, as such is applicable to all the MINs which show the generality of the algorithm. Further, the main tasks in the algorithms are the dis-jointing process and the path enumeration process. Path enumeration by using the existing algorithms provides a worst case running time of N^3, where N is the number of nodes in the probabilistic graph. On the other hand the disjointing process requires a worst case running time of N^2. So, using the concept of analyzing the algorithm, the path enumeration dominates all other operations including the dis-jointing process. Hence, the worst case running time of the proposed algorithm is $O(N^3)$ i.e. cubic in nature.

3. A COMPARATIVE STUDY OF THE PROPOSED ALGORITHM WITH AN EXISTING TECHNIQUE [23]

Table 1 gives a comparative value of reliability computed by the proposed method with the analytical method [23] for the following networks: Shuffle exchange network (SEN),

Table 1
Comparison of the Values of Terminal Reliability of MINs Computed by Proposed Method with an Existing Method

Switching probability	*TR of SEN*		*TR of SEN+*		*TR of SEN+2*	
0.99	0.9865	0.9702	0.9801	0.9747	0.9508	0.9243
0.98	0.9605	0.9411	0.9667	0.9588	0.9067	0.8567
0.96	0.9025	0.8647	0.9320	0.9159	0.8165	0.7426
0.95	0.8679	0.8547	0.9109	0.8939	0.7630	0.6946
0.94	0.8507	0.8305	0.8890	0.8716	0.6930	0.6518
0.92	0.8056	0.7756	0.8420	0.8254	0.6098	0.5792
0.90	0.7502	0.7290	0.8129	0.7807	0.5409	0.5209

Extra stage shuffle exchange network (SEN+) and Extra stage network with two extra stages (SEN + 2).

Table 1 reveals that the proposed method provide more accurate value of reliability than its counterpart.

4. Evaluation of Reliability of Some Important Fault-tolerant MINs

The residual broadcast reliability of some important fault-tolerant MINs viz. Extra Stage Cube (ESC) [1], Extra stage shuffle exchange network (SEN+) [23], Multi path Chained Baseline Network (MPN) [1] and New four Tree Multistage Interconnection Network (NFT) [24] are evaluated and compared by the proposed algorithm in different environments.

Calculation of reliability of links:

With a given, the reliability of links can be computed using the Exponential model i.e.

$$p(t) = e^{\left(-\int_0^t [\lambda(u)du]\right)} \tag{1}$$

$$p(t) = e^{-\lambda t}$$

Using equation (1), the reliability of links can be computed with respect to a given time t at different link failure rates. The terminal reliability of different fault-tolerant MINs can be computed by using the proposed method. Table 2, 3 and Fig. 2 present comparative results of terminal reliabilities of five different fault-tolerant MINs. From all these results, it can be concluded that, the MINs can be arranged in the following manner with respect to their terminal reliability values.

$$\text{NFT} > \text{MPN} > \text{ESC} > \text{SEN+} > \text{SEN} \tag{2}$$

Table 2
Comparison of Terminal Reliability of MINs when Link Failure Rate $\lambda = 0.001$

Time in Hours	*Terminal Reliability*			
	ESC	*SEN+*	*MPN*	*NFT*
100	0.8818	0.8113	0.9000	0.9705
200	0.7478	0.6483	0.7921	0.8916
300	0.6173	0.5119	0.6780	0.7834
400	0.4998	0.4005	0.5656	0.6649
500	0.3987	0.3109	0.4616	0.5494
600	0.3145	0.2399	0.3699	0.4444
700	0.2457	0.1841	0.2920	0.3536
800	0.1906	0.1407	0.2277	0.2777
900	0.1470	0.1071	0.1758	0.2157
1000	0.1127	0.0813	0.1347	0.1662

Table 3
Comparison of Terminal Reliability of MINs with Respect to λ at time $t = 200$ hrs.

Value of	*Terminal Reliability*			
λ	*ESC*	*SEN+*	*MPN*	*NFT*
0.001	0.7478	0.6483	0.7921	0.8916
0.002	0.4998	0.4005	0.5656	0.6649
0.003	0.3145	0.2399	0.3699	0.4444
0.004	0.1906	0.1407	0.2277	0.2777
0.005	0.1127	0.0813	0.1347	0.1662

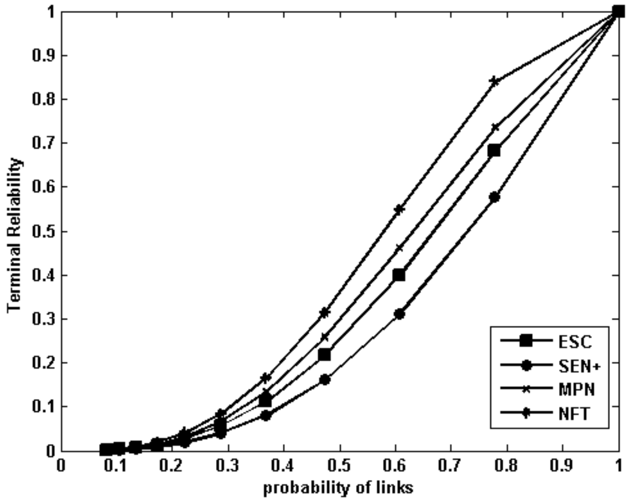

Figure 2: Comparison of Terminal Reliability of MINs with Respect to Different Probabilities when $\lambda = 0.0025$

CONCLUSION

The proposed method is simple still computed more accurate terminal reliability of MINs in comparison with its counterpart. It is also general and as such is applicable to all the MINs. Reliability of ESC, SEN+, MPN and NFT are calculated by the proposed methods. The results shows that NFT is having the highest value of reliability than other said MINs.

REFERENCES

[1] G. B. III Adams, D. P. Aggarwal, H. J Siegel, A Survey and Comparison of Fault-tolerant Multistage Interconnection Network, *IEEE Computers*, (1987), 14–27.

[2] J. Yang, X. Su and K. Ping, A Module Design of Rearrangeable Nonblocking Double Omega Optical Network using Binary Optics Elements, *Optics and Laser*, Elsevier, **40**, (2008), 756–767.

[3] G. B. Adams-III and H. J. Siegel, The Extra-stage Cube: A Fault-tolerant Interconnection Network for Super Systems, *IEEE Trans. Computer,* **C-31**(5), (1982), 443–454.

[4] S. Arige, E. V. Prasad, C. N. Reddy, Permutation Capability and Connectivity of Enhanced Multistage Interconnection Network (E-MIN). *International Conference on Advanced Computing and Communications*, (2007), 8–11.

[5] K. V. Arya, R. K. Gosh, Designing a New Class of Fault Tolerant Multistage Interconnection Networks. *Journal of Interconnection Network*, **6**(4), (2005), 361–382.

[6] L. N. Bhuyan, Q. Yang and D. P. Agrawal, Performance of Multiprocessor Interconnection Networks, *IEEE Comput.*, **2**, (1989), 25–37.

[7] J. T. Blake and K. S. Trivedi, Reliability Analysis of Interconnection Networks using Hierarchical Composition, *IEEE Trans. Reliab.,* **38**, (1989), 111–120.

[8] J. Kim, C. R. Das, W. Lin and T. Y. Feng, Reliability Evaluation of Hypercube Multicomputes, *IEEE Trans, Reliab.,* **38**, (1989), 121–129.

[9] X. Cheng and O. C. Lbe, Reliability of a Class of Multistage Interconnection Network, *IEEE Trans. Parallel and Distributed System,* **3**(2), (1992), 241–246.

[10] C. J. Colbourn, J. S. Devitt, D. D. Harms and M. Kraetzl, Assessing Reliability of Nultistage Interconnection Networks, *IEEE Trans. Comput.,* **42**(10), (1993), 1207–1221.

[11] Gunawan, Performance Analysis of a Multistage Interconnection Network System Based on a Minimum Cut Set Method, *International Journal of Performability Engineering*. **4**(2), (2008), 111–120.

[12] A. K. Katangur, S. Akkaladevi, Y, Pan, Analyzing the Performance of Optical Multistage Interconnection Networks with Limited Crosstalk. *Cluster Computing* **10**(2), (2007), 241–250.

[13] B. L. Menezes, U. Bakhru, New Bounds on the Reliability of Augmented Shuffle-Exchange Networks. *IEEE Transactions on Computers*, **44**(1), (1995), 123–129.

[14] S. Rai and Y. C. Oh, Tighter Bounds on Full Access Probability in Fault-tolerant MINs, *IEEE Trans. Parallel and Distributed System,* **10**, (1999), 327–335.

[15] S. Sharma, P. K. Bansal, A New Fault Tolerant Multistage Interconnection Network , *IEEE TENCON'02*, **1**, (2002), 347–350.

[16] J. Sengupta, P.K. Bansal "Performance of Regular and Irregular Dynamic MINs, *IEEE TENCON 99* , **1**, (1999), 427–430.

[17] A. K. Somani, N. H. Vaidya, Understanding Fault-Tolerance and Reliability. *Computer*, **30**(4), (1997).

[20] C. R. Tripathy, S. Patra, R. B. Misra and R. N. Mahapatra, Reliability Evaluation of Multistage Interconnection Networks with Multistate Elements, *Microelectron. Reliab. - An International Journal,* **36**(3), (1996), 423–428.

[21] Varma and C. S. Raghavendra, Reliability Analysis of Redundant Path Interconnection Networks, *IEEE Trans, Reliab.,* **38**, (1989), 130–137.

[22] Y. Yang, J. Wang, Y. Pan, Permutation Capability of Optical Multistage Interconnection Network, *Journal of Parallel and Distributed Computing*, (2002), 60.

[23] I. Gunawan, Reliability Analysis of Shuffle-exchange Network Systems, *Reliability Engineering and System Safety*, **93**, (2008), 271–276.

[24] S. Sharma, P. K. Bansal and K. S. Kahlon, On a Class of Multistage Interconnection Network in Parallel Processing, *IJCSNS International Journal of Computer Science and Network Security*, **8**(5), (2008).

[25] I. Gunawan, Redundant Paths and Reliability Bounds in Gamma Networks, *Applied Mathematical Modelling*, **32**, (2008), 588–594.

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