International Journal of Information Technology and Knowledge Management
January-June 2008, Volume 1, No. 1, pp. 139-145

FRACTIONAL FOURIER AND COSINE TRANSFORMS IN IMAGE COMPRESSION

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ABSTRACT

In this paper, fractional Fourier transform (FRFT) and fractional Cosine Transform (FRCT) are used for image compression and are compared with conventional Fourier Transform (FT) and Cosine Transform (CT). It has been observed, that a good fidelity of decompressed image can be achieved by taking different fractional values of the transforms. FRFT provides better mean square error (MSE) and peak signal to noise ratio (PSNR) for the same compression ratio (CR) as compared to FRCT while preserving image quality.

Keywords: Fractional Fourier transform, Fractional Cosine transform, Image Compression.

1. INTRODUCTION

Large amount of research work has been carried out on data compression in general and image compression in particular, leading to efficient compression algorithms. The rapid growth of digital imaging applications, including desktop publishing, multimedia, teleconferencing, and high-definition television (HDTV) has increased the need for effective and standardized image compression techniques[1]. In addition, it is the natural technology for handling the increased spatial resolutions of today's imaging sensors and evolving broadcast television standards. The purpose of image compression is to achieve a very low bit rate representation, for example. MPEG-4[2] aims at 64K bits per second while preserving a high visual quality of decompressed images. The significant feature of fractional Fourier and Cosine domain image compression benefits from its extra degree of freedom that is provided by their fractional orders.

The FRFT, which is a generalization of the ordinary Fourier transform (FT), was introduced 75 years ago, but only in the last two decade it has been actively applied in signal processing, optics and quantum mechanics. It gives a more complete representation of the signal in phase space and enlarges the number of applications of the ordinary FT[3]. In addition to the FT, the cosine transform (CT), which are based on half-range expansion of a function over cosine basis function, are also important tools in signal processing. Despite some lack of elegance in their properties compared to the FT, the CT has their own areas of applications. The idea of fractionalization of the CT was proposed in[4]. The real part of the FRFT kernel was chosen as the kernels for a FRCT as in the case of CT where real part of FT is chosen as CT kernel. Thus, FRFT and FRCT with parameter ‘a’ are finding its place in many applications where FT and CT are found to be useful like image processing, beam forming, noise removal and signal restoration. In section 2, the definition of FRFT and FRCT presented. Section 3, presents image compression using FRFT/FRCT. The paper concludes with numerical results and conclusions.

2. FRACTIONAL FOURIER AND COSINE TRANSFORMS

The FRFT $F^{\alpha}(u)$ of a function $f(x)$ is defined as[3]

$$F^{\alpha}(u) = R_F^{\alpha}[f(x)](u) \tag{1}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} k_{\alpha}(x,u) f(x) \exp\left(-jux/\sin\alpha\right) dx$$

where

$$k_{\alpha}(x, u) = \frac{\exp\left(j\frac{1}{2}\alpha\right)}{\sqrt{j \sin\alpha}} \exp\left[\frac{1}{2} j(x^2 + u^2) \cot\alpha\right] \tag{2}$$

For $\alpha = \pi/2$, for which $k_{\pi/2}(x, u) = 1$, we have the conventional FT, while for $\alpha \to 0$ we have the identity transformation: $F^0(x) = f(x)$. The FRFT of an even function is even, while the FRFT of an odd function is odd. Consider one-sided function $f(x)$, with $f(x) = 0$ for $x < 0$, and define the fractional CT as

$$F_c^{\alpha}(u) = R_c^{\alpha}[f(x)](u)$$

$$= R_F^{\alpha}[f(x)] + f(-x)](u)$$

$$= F^{\alpha}(u) + F^{\alpha}(-u)(u \geq 0)$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} k_{\alpha}(x,u) f(x) \cos\left(ux/\sin\alpha\right) dx \tag{3}$$

which reduce to the normal CT [4] for $\alpha = \pi/2$. In this way it is clear that, R_c^{α} is related to the even part of R_F^{α}, to determine the FRCT of a causal, one-sided function, $f(x)$, determine the FRFT of the evenly extended two-sided function $f(x) + f(-x)$.

The one-dimensional FRFT and FRCT[5] are useful in processing single-dimensional signals such as speech waveforms. For analysis of two-dimensional (2D) signals such as images, a 2D version of the FRFT and FRCT needed. For an M´N matrix, the 2D FRFT/FRCT are computed in a simple way: The 1D FRFT and FRCT[6, 7] is applied to each row of matrix and then to each column of the result. In the case of the two-dimensional

FRFT and FRCT[8, 9] we have to consider two angles of rotation $\alpha = a\pi/2$ and $\beta = b\pi/2$. If one of these angles is zero, the 2D transformation kernel reduces to the 1D transformation kernel.

3. IMAGE COMPRESSION USING FRFT AND FRCT

In image processing, an important part is the compression. This means reducing the dimensions of the images, to a level that can be easily used or processed. Image compression using transform coding yields extremely good compression, with controllable degradation of image quality[11]. By adjusting the cutoff of the transform coefficients, a compromise can be made between image quality and compression factor. To exploit this method, an image is first partitioned into non-overlapped $n \times n$ sub images as shown in Fig. 1. A 2D-FRFT/2D-FRCT is applied to each block to convert the gray levels of pixels in the spatial domain into coefficients in the frequency domain. The coefficients are normalized by different scales according to the cutoff selected. The quantized coefficients are rearranged in a zigzag scan order. At decoder simply inverse process of encoding by using inverse 2D-FRFT/2D-FRCT is performed. The PSNR value used to measure the difference between a decoded image $\hat{f}$ and its original image f is defined as follows. In general, the larger PSNR value, the better will be decoded image quality.

$$MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [\hat{f}(i, j) - f(i, j)]^2 \tag{4}$$

$$PSNR = 10 \log_{10} \left[\frac{255 \times 255}{MSE} \right] dB \tag{5}$$

where $M \times N$ is the size of the images, $\hat{f}(i, j)$ and $f(i, j)$ are the matrix elements of the decompressed and original images at (i, j) pixel.

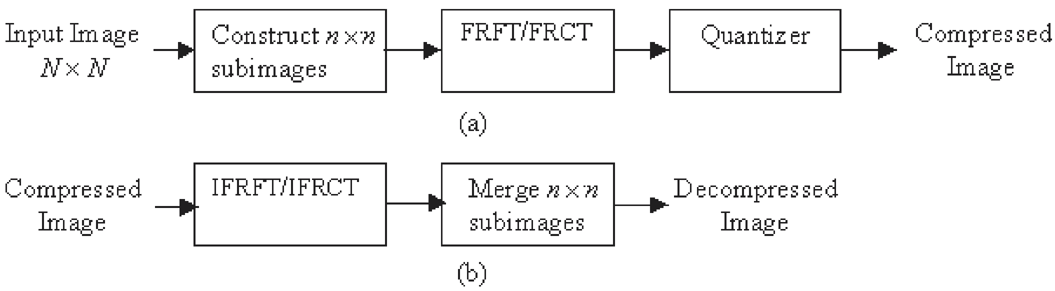


Figure 1: FRFT/FRCT Compression Model: (a) Encoder; (b) Decoder

4. Results and Conclusion

Numerical simulations were performed. In order to compare the image compression using FRFT and FRCT, image compression experiments using FRFT/FRCT are conducted on three natural images i.e. Lena, Rice, Barbara with 256 × 256 pixels. Fig 2, 3, show the results of earlier mentioned three images using FRFT/FRCT for same *CR* = 25:1 and with different fractional orders '*a*'. It is clear that at '*a*' = 0.1 image quality is very poor for all images but as '*a*' increases, image quality improves. For $a \in [0.9\text{-}1]$, both FRFT and FRCT gives optimum domain for which mean square error (MSE) is very less, peak signal to noise ratio (PSNR) is very high and better decompressed image is retained.

The curves of PSNR versus the changes of fractional order '*a*' for earlier three images have been calculated and depicted in Fig. 5(a)-(c). It is clear from Fig. 5(a) that at '*a*' = 0.1, error is too large but as '*a*' increases there is an decrease in MSE and correspondingly there is increase in PSNR. In Lena image using FRFT, MSE can decrease up to 90.48, PSNR increase to 27.47 at '*a*' = 0.94 but in FRCT, MSE decreases to 184.23, PSNR increase to 24.45 at '*a*' = 0.96 for same *CR* of 25:1. At '*a*' = 1 FRFT/ FRCT approaches to conventional FT/CT where again image quality degrades as compare to its fractional domain. Similarly for other two images FRFT gives better image quality as compared to FRCT.

Table 1 shows the comparison of FRFT and FRCT and gives the optimum domain for all images. It is clear from table1 and 2 that FRFT gives MSE which is nearly half as that of MSE in FT ('*a*' = 1) domain but in FRCT the improvement in MSE is very small as compared to its CT domain. It is clear that these newly proposed transforms give better results as compared to existing transforms.

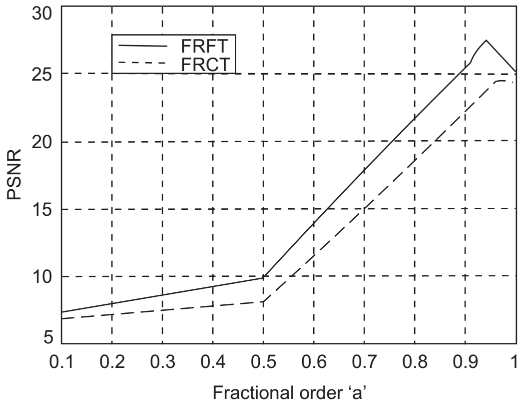
Table 1
Comparison of FRFT and FRCT for Different Images for *CR* = 25

	FRFT			*FRCT*		
IMAGES	*Optimum domain 'a'*	*MSE*	*PSNR (dB)*	*Optimum domain 'a'*	*MSE*	*PSNR (dB)*
LENA	0.94	90.48	27.47	0.96	184.23	24.45
RICE	0.94	19.73	34.02	0.97	75.78	28.25
BARBARA	0.95	84.15	27.75	0.96	125.38	26.06

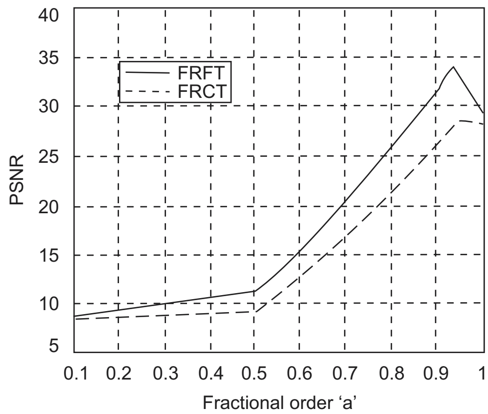
Table2
Comparison of FT and CT for Different Images for *CR* = 25

	FT ('a' = 1)		*CT ('a' = 1)*	
IMAGES	*MSE*	*PSNR (dB)*	*MSE*	*PSNR (dB)*
LENA	155.62	25.12	191.77	24.22
RICE	57.61	29.44	79.87	28.04
BARBARA	137.91	25.65	138.43	25.63

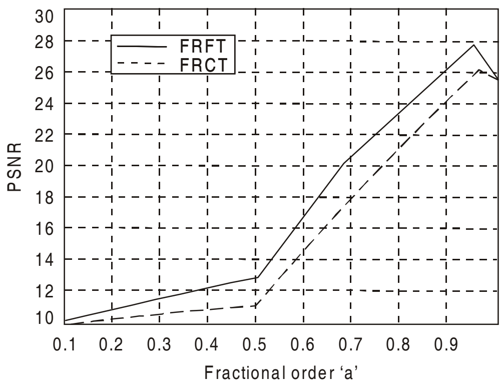
Figure 2: Simulation Results of the Different Images with 256 × 256 Pixels with *CR* = 25 for Varying '*a*'



(a) PSNR vs. Fractional Orders for Lena Image



(b) PSNR vs. Fractional Orders for Rice Image



(c) PSNR vs. Fractional Orders for Barbara Image

Figure 3: Simulation Results of Different Images with 256 × 256 Pixels with *CR* = 25

In conclusion, a technique for image compression using FRFT/FRCT makes the full use of the additional parameter '*a*' to achieve better PSNR, less MSE and high compression ratio. For same amount of compression FRFT gives better image quality as compared to FRCT, moreover both FRFT and FRCT are better as compared to conventional FT & CT.

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