International Journal of Information Technology and Knowledge Management
July-December 2012, Volume 5, No. 2, pp. 433-438

INVERSE RESPONSE COMPENSATION BY ESTIMATING PARAMETERS OF A PROCESS COMPRISING OF TWO FIRST ORDER SYSTEMS

Mandeep Singh[1] and Abhay Sharma[2]

Abstract: An inverse response in a system occurs when the initial response of an output variable occurs in the opposite direction to that of the steady state value. The classical example of this behaviour can be seen in a boiler with two different first order systems. This paper helps to estimate the transfer function parameters of the process from its unit step response. Classical method for this estimation involves simultaneous solution of three non-linear equations containing exponential terms. Most of the available tools fail to achieve this solution. The paper proposes a novel iterative method to overcome this limitation.

Keywords: inverse response, transfer function, non-linear equations, parameter estimation

1. INTRODUCTION

Whenever a controlled variable encounters two (or more) competing dynamic effects with different time constants from the same manipulated variable, the resulting composite dynamic behaviour of this process can exhibit a troublesome inverse response or a large overshoot response. In process control, inverse response (IR) processes can be commonly encountered and classical examples are illustrated in textbooks (Tyner and May(1968) [1], Vidyasagar (1985)[2]). The important characteristic of processes with inverse response is that the process transfer function has odd number of zeroes on the right hand side of origin [3]. IR processes are frequently encountered in process control like drum boiler, distillation column and boost converter [4], [5]. Scali *et al* [6] and Zhang *et al* [3] proposed an analytical design for IR processes without dead time. This inverse response problem affects the achievable closed-loop performance because the controller operates on wrong sign information in the initial time of the transient [7].

Let us take two first order systems with opposing gains. The problematic situation is the result of two opposing dynamics. With the given opposite first order systems let us study the combined process control of a process.

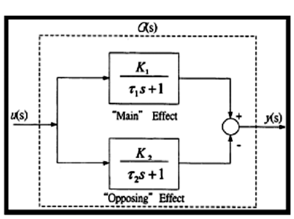

Figure 1: Process Block Diagram for IR Process

[1,2] Department of Electrical and Instrumentation Engineering, Thapar University, Patiala, India,
[1]E-mail: *mandy_tiet@yahoo.com*, [2]*s.abhay24@gmail.com*

$$G(s) = \frac{y(s)}{u(s)} \tag{1}$$

$G(s)$ refers to the overall transfer function of the process

$$G(s) = G_1(s) + G_2(s)$$

$G_1(s)$ and $G_2(s)$ are parallely coupled opposing first order systems as shown in figure 2.

$$G_1(s) = \frac{K_1}{\tau_0 s + 1} \text{ and } G_2(s) = -\frac{K_2}{\tau_2 s + 1}$$

$$G(S) = \frac{K_1}{\tau_1 s + 1} - \frac{K_2}{\tau_2 s + 1} = \frac{(K_1\tau_2 - K_2\tau_1)s + K_1 - K_2}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{2}$$

K_1 refers to the proportionality gain constant of initial first order system. K_2 refers to the proportionality gain constant of other first order system. τ_1 and τ_2 are the time constants of the two first order systems respectively. $y(s)$ and $u(s)$ are the final output and step input respectively as in figure 1. We can get the zeroes of the transfer function by equating the numerator of the transfer function to zero. We get the following equation

$$(K_1\tau_2 - K_2\tau_1)s + K_1 - K_2 = 0, S = -\frac{K_1 - K_2}{(K_1\tau_2 - K_2\tau_1)} \tag{3}$$

This value of s gives the value of zero on the right hand side of the origin. We have inverse response when $\frac{\tau_1}{\tau_2} > \frac{K_1}{K_2} > 1$, i.e., initially process 2, which reacts faster than process 1, dominates the response of the overall system, but ultimately process 1 reaches a higher steady-state value than process 2 and forces the response of the overall systems in the opposite direction as shown in Figure 2.

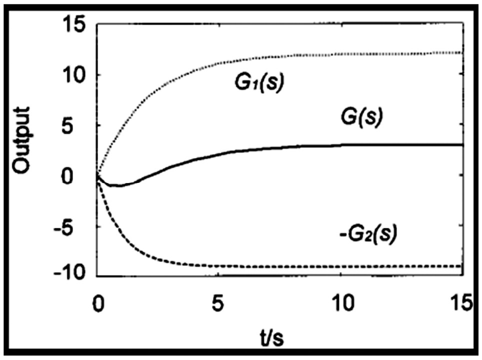

Figure 2: Inverse Response Curve

In this context, inverse response appears due to competing effects of slow and fast dynamics. In concrete terms, it appears when the slower process has higher gain, the open-loop response to a step input presents undershoot, the more pronounced the more it increases the value of zero.

2. PROBLEM DEFINITION

The problem of inverse response has been a challenging task for the process engineers for a long time. Basically, if process parameters are estimated then a compensator may be defined for it. We estimate the parameters such that the process follows first order slowly and then reaches a steady value [8]. The main problem is that Classical method for this estimation involves simultaneous solution of three non-linear equations containing exponential terms. Most of the available tools fail to achieve this solution. The paper proposes a novel iterative method to overcome this limitation.

3. METHOD

This section would be helpful in evaluating the process parameters by mathematically studying the graph. For solving the equations we consider no dead time delay. From the graph shown in figure 3, we can see that its minima occurs at time $t = t_1$. The function holds a zero value at time $t = t_2$ and from observation it is clearly understood that the plot has its derivative as zero at time $t = t_1$. The function reaches its steady state value after some time. The derivative of the function remains zero at steady state. The following equation (4) expresses the output function in time domain which was earlier expressed in frequency domain.

$$Y(t) = K_1(1 - e^{-t/\tau_1}) - K_2(1 - e^{-t/\tau_2}) \qquad (4)$$

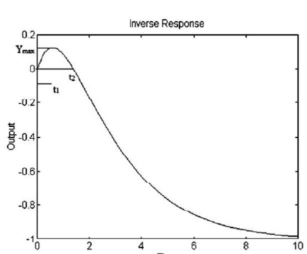

Figure 3: Graphical Representation of Output Function Showing Required Parameters

The derivative of the plot in figure 3 is given by the equation

$$\frac{dy}{dt} = \frac{K_1}{\tau_1} * e^{-\frac{t}{\tau_1}} - \frac{K_1}{\tau_2} * e^{-\frac{t}{\tau_2}} \qquad (5)$$

The value at the function at its minima is given as

$$Y_{\min} = K_1(1 - e^{-t_1/\tau_1}) - K_2(1 - e^{-t_1/\tau_2}) \qquad (6)$$

Derivative of the function holds zero value at its minima

$$\frac{dy}{dt} = \frac{K_1}{\tau_1} * e^{-\frac{t_1}{\tau_1}} - \frac{K_1}{\tau_2} * e^{-\frac{t_1}{\tau_2}} = 0 \qquad (7)$$

The value of the function is zero at time ($t = t_2$. So this can be written as

$$Y(t_2) = K_1(1 - e^{-t_2/\tau_1}) - K_2(1 - e^{-t_2/\tau_2}) = 0 \qquad (8)$$

The final equation can be obtained at the steady state value where the function holds a constant value. This can be done by putting a limit on $Y(t)$ to get a desired value at infinite time. It is stated as below

$$\lim_{t \to \infty} Y(t) = K_1 - K_2 \qquad (9)$$

This is because the exponential term vanishes as time approaches infinity. So we are just left with the difference of the proportionality gain constants of the two functions i.e. $K_1 - K_2$.

There can also be a case that the final output follows initial first order system rather than other, in that case equations would become

$$Y_{\max} = K_1(1 - e^{t_1/\tau_1}) - K_2(1 - e^{t_1/\tau_2}) \qquad (10a)$$

$$\frac{dy}{dt} = \frac{K_1}{\tau_1} * e^{\frac{t_1}{\tau_1}} - \frac{K_1}{\tau_2} * e^{\frac{t_1}{\tau_2}} = 0 \qquad (10b)$$

$$Y(t_2) = K_1(1 - e^{t_2/\tau_1}) - K_2(1 - e^{t_2/\tau_2}) = 0 \qquad (10c)$$

$$\lim_{t \to \infty} Y(t) = K_1 - K_2 \qquad (10d)$$

The above stated four equations (10*a*-10*d*) are sufficient to solve four variables (K_1, K_2, τ_1, τ_2). The remaining variables which are t_1, t_2, $Y_{\max}$ or $Y_{\min}$, $\frac{dy}{dt}$ can be estimated graphically from the figure 3 to get an approximate value of the unknown variables. The dynamic characteristics of the model as shown by above equations (10*a*-10*d*) are evaluated in MATLAB software to get the value of the unknown variables. The solution cannot be obtained by using the **'solve'** command because the unmodified form of the three exponential equations does not return any solution using this command. This kind of problem cannot be tackled in MATLAB and so we use an alternative approach to it.

Firstly we use substitution technique in which we replace the value of K_1 by K_2 in all equations using equation (9) as $\lim_{t\to\infty} Y(t)$ holds a given unique value. Now we are left with three unknown parameters K_2, τ_1 and τ_2 and three simultaneous equations. Since the MATLAB shows an error in solving three nonlinear algebraic equations, we use two non-algebraic equations whose result is always unique. For the third variable we use iterative method. The third variable is chosen to be τ_1 because it is in exponential form and any small variation in its value will cause a large change in the value of output function thereby limiting the number of considerations. The basic approach used is 'LEAST SQUARE ERROR (L.S.E.)'. For an instance, suppose we assume τ_1 to be a variable and assume its value to be 1, then we would compute the value of unknown variables and plot the corresponding curve. Since the initial curve was given to us, we would now calculate the difference of plotted curve to the initial curve at each point and take its square. This L.S.E. method is done for a large number of values of τ_1. The L.S.E. curve is now plotted and by number of experiments we observe a global minimum for different given iterations. The optimized value is actually the one occurring at the minima of the L.S.E. curve. User's concern is to identify the minima boundary limits and then using simple iterative method we can approximate the position of minima thereby evaluating the corresponding unknown variables. The error will reflect the accuracy of the solution. The following Matlab programme throws some light on the solution:

s=solve ('((K_2+$Y_{steady\text{-}state}$)/n)*exp(–t_1/n) – (K_2/τ_2)*exp(–t_1/τ_2) ',' (K_2+$Y_{steady\text{-}state}$)*(1–exp(–t_2/n)) – K_2*(1–exp(–t_2/τ_2))')

K_2=s.K_2

τ_2=s.τ_2

t=[1:100]

y(t)=(K_2+ $Y_{steady\text{-}state}$)*(1–exp(–t/n)) – K_2*(1–exp(–t/τ_2))

h=(y(t)–m(t))

l=transpose(h)

q=h*l

In the above programme 'solve' command is helpful in evaluating the equations written in it. n is trial value given to τ_1, m(t)gives the value of the given plot at intervals defined by t(t=[1:100]) and q is the value of L.S.E.

A different programme is used when the given curve has its positive maxima. The following Matlab programme explains this:

s=solve ('–((K_1– $Y_{steady\text{-}state}$)/τ_2)*exp(–t_1/τ_2) + (K_1/n)*exp(–t_1/n) ',' –(K_1–$Y_{steady\text{-}state}$)*(1–exp(–t_2/τ_2)) + K_1*(1–exp(–t_2/n))')

τ_2=s.τ_2

K_1=s.K_1

t=[1:100]

y(t)=–(K_1– $Y_{steady\text{-}state}$)*(1–exp(–t/τ_2)) + K_1*(1–exp(–t/n))

h=(y(t)–m(t))

l=transpose(h)

q=h*l

4. RESULTS AND DISCUSSIONS

This method of parametric estimation of Inverse Response is primarily dependent on the unit step of the process. For this purpose, we have to simulate this response using MATLAB SIMULINK. The process defined by equation (4) is simulated for its response to the step input. The values of K_1, K_2,τ_1 and τ_2 are chosen for six different cases and the transfer function so formed is shown in table 1. Care is taken while chosing these values so as to follow the condition $\frac{\tau_1}{\tau_2} > \frac{K_1}{K_2} > 1$. After getting the values of t_2, t_1 and Y_{min} or dy/dx and y_{max} from the simulated graph, the proposed method is applied for. These values are reported in table 1. The resulting value of estimated K_1, K_2, τ_1 and τ_2 are also shown in table 2. It is clearly seen that the errors are only on acount of the manual measurements done for of t_2, t_1, Y_{min} and y_{max}. The simulated graphs are shown in figure 4(*a-f*).

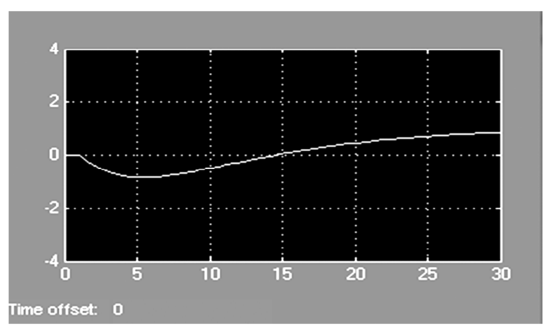

Figure 4(*a*): Assumed Process Curve

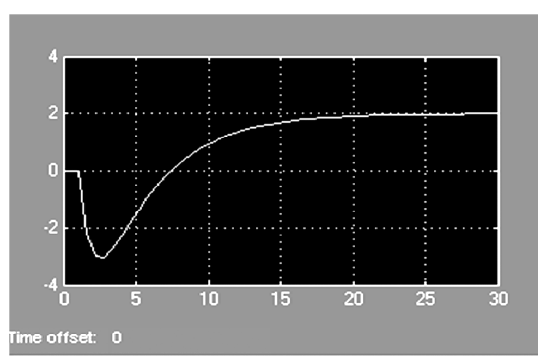

Figure 4(*b*): Assumed Process Curve

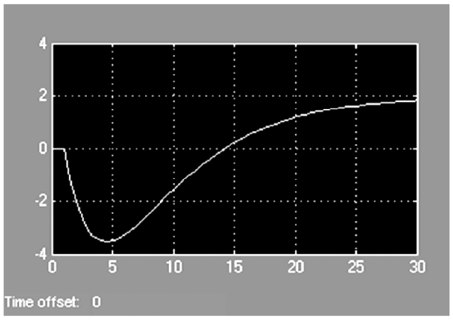

Figure 4(*c*): Assumed Process Curve

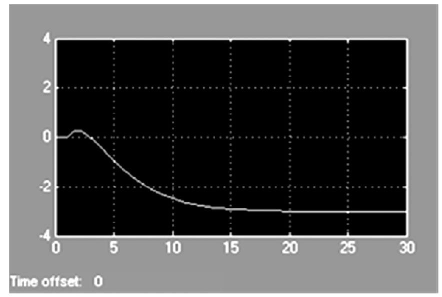

Figure 4(*d*): Assumed Process Curve

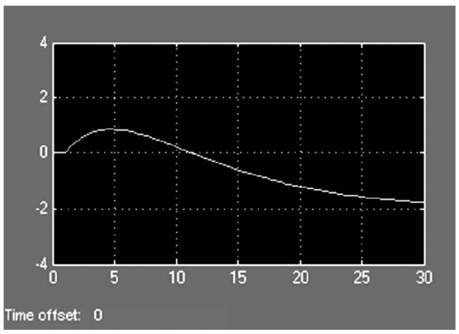

Figure 4(*e*): Assumed Process Curve

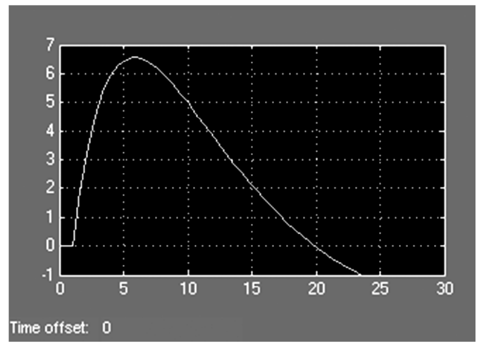

Figure 4(*f*): Assumed Process Curve

Table 1
Results of Graphical Estimation

Sl no	*Original Transfer Function*	*Minima of L.S.E lies between*	*Value of t_1 in sec*	*Value of t_2 in sec*	*Value of Y_{min}*	*Value of Y_{max}*
1.	$\frac{12}{7s+1}-\frac{11}{5s+1}$	[6, 8]	4.4	13.55	–0.8376	1
2.	$\frac{10}{4s+1}-\frac{8}{s+1}$	[3, 5]	1.6	6.41	–3.088	2
3.	$\frac{20}{6s+1}-\frac{18}{3s+1}$	[5, 7]	3.6	13.18	–3.5547	2
4.	$\frac{10}{2s+1}-\frac{13}{3s+1}$	[1, 3]	0.8	1.83	–3	0.25
5.	$\frac{15}{5s+1}-\frac{17}{7s+1}$	[4, 5]	3.6	10.25	–2	0.86
6.	$\frac{32}{4s+1}-\frac{35}{8s+1}$	[3, 5]	4.8	18.94	–3	6.57

Table 2
Results of Parametric Estimation

Serial number	*Original Transfer Function*	*Value of K_1 and %error*	*Value of K_2 and %error*	*Value of τ_1 and %error*	*Value of τ_2 and %error*	*Value of L.S.E.*	*Referred Figure*
1.	$\frac{12}{7s+1}-\frac{11}{5s+1}$	11.8803, – 1.6167%	10.8803, – 1.08818%	7.1, 1.428%	5.0962, 0.01962%	0.0076	4a

Table Cont'd

Table 2 Cont'd

2.	$\frac{10}{4s+1}-\frac{8}{s+1}$	10.00583, 0.05%	8.00583, 0.07%	3.96, –1%	1.069, 6.9%	0.063	4b
3.	$\frac{20}{6s+1}-\frac{18}{3s+1}$	20.4974, 2.487%	18.4974, 2.7663%	6.01, 0.167%	3.161, 5.367%	0.24	4c
4.	$\frac{10}{2s+1}-\frac{13}{3s+1}$	9.9561, – 0.439%	12.9561, – 0.33769%	2.0128, 0.64%	2.98, – 0.667%	0.017	4d
5.	$\frac{15}{5s+1}-\frac{17}{7s+1}$	14.6468, – 2.35%	16.6468, –2%	4.95, –1%	6.9967, – 0.33%	0.00077	4e
6.	$\frac{32}{4s+1}-\frac{35}{8s+1}$	31.6764, –1.011%	34.6764, – 0.9247%	3.94, – 1.5%	8.01, – 0.125%	0.079	4f

The above shown results are for model processes whose output functions are shown by figures (4*a*-4*f*). Delay in each plot is assumed zero and graphical estimation has played an important role in the estimation of process parameters. The result has been possible solely due to the presence of single global minima otherwise iterative method would have possibly failed. The results show the error occurred in each case limited to the estimation technique of each user.

5. CONCLUSION

This paper represents a novel method for the estimation of inverse response parameters of a process model. Two opposing cases of first order processes are considered and the given Matlab software helps in estimating the process parameters of the curve for the unit step applied. It should be noticed that process is ideal without any process delay. This paper shows that three exponential equations and a linear equation has been solved which was not possible with the classical method and the earlier publication related to parametric estimation of inverse response process comprising of first order system and a pure capacitive system [9] has been crucial in understanding of process model and its dynamics. The result is obtained by least square error and iterative method and the limitation due to non-linear equations has been eliminated.

6. FUTURE SCOPE

The procedure described above gives a highly reliable method which is easy to use and implement. The estimating procedure is suitable for application in industrial environment because of its simplicity. It has its application in dead time compensation and inverse response compensation [10]. The results show that it can be applied in the estimation of large number of processes giving better estimation in less time. This method is helpful in designing a suitable compensator, by which the fine tuning of the PID (Proportional Integral and Derivative) controller [11]. This is possible only if the process parameters are known to us.

REFERENCES

[1] Tyner, M.; May. F.P., "Process Engineering Control", *Ronald Press: New York*, 1968.

[2] Vidyasagar M., "On Undershoot and Non-minimum Phase Zeros", *IEEE Trans. Autom. Control 1986*, **31**, 440-441.

[3] Zhang W.; Xu. X.; Sun. Y., "Quantitative Performance Design for Inverse-response Processes", *Ind. Eng. Chem. Res*, 39, 2056-2061, 2000.

[4] Shinskey F.G., "Process Control System", *2nd ed.; McGraw-Hill Book Company: New York*, 1979.

[5] Arulselvi S., Uma G., Chidambaram M., "Design of PID Controller for Boost Converter with RHS Zero", *Conf. Proc. IPEMC Int. Power Electron. and Motion Control Conf*, pp. 532-537, 2004.

[6] Scali C.; Rachid A., "Analytical Design of Proportional-Integral-Derivative Controller for Inverse Response Processes", *Ind. Eng. Chem.Res.*, **37**, pp. 1372-1379, 1998.

[7] Skogestad S.; Postlethwaite I., "Multivariable Feedback Control", Wiley, 1997.

[8] Iinoia K.; Alperter R.J., "Inverse Response in Process Control", *Ind.Eng. Chem.*, **54**, pp. 39-43, 1962.

[9] Singh Mandeep; Saksena Dhruv, "Parametric Estimation of Inverse Response Process Comprising of First Order System and a Pure Capacitive System", *International Journal of Information Technology and Knowledge Management,* **5(2)**, June 2012 (in press).

[10] Stephanopoulos.G., "Chemical Process Control: An Introduction to Theory and Practice", *Prentice-Hall*, 1984.

[11] Waller K.T.V.; Nygårdas C.G., "On Inverse Response in Process Control", *Ind. Eng. Chem. Fundam*, **14**, pp. 221-223, 1975.