

PARAMETRIC ESTIMATION OF INVERSE RESPONSE PROCESS COMPRISING OF FIRST ORDER SYSTEM AND A PURE CAPACITIVE SYSTEM

Mandeep Singh¹ and Dhruv Saksena²

Abstract: The classical example of an integrating process with inverse response is liquid level control in a simple drum boiler system. It is noticed that initially the response is in opposite direction to where it eventually ends up. The swelling behaviour may be modelled as the net result of opposite effects of a pure integrator and a first order system without any time delay. This paper helps to estimate the transfer function parameters of the process from its unit step response. The procedure is based on graphical evaluation of the given process with suitable mathematical equations.

Keywords: Integrating process, drum boiler, swelling behaviour, inverse response, parametric estimation

1. INTRODUCTION

An inverse response occurs due to the presence of right half plane zeroes or when the initial response is in the opposite direction with respect to the ultimate steady-state value. The common 'boiler swell' problem is an example of it [1]. The essential characteristics of this process is the presence of at least one zero on the right hand plane. The inverse response processes are frequently encountered in boilers, distillation processes and boost converter [2].

Consider a simple drum boiler in which the level is controlled by manipulating the boiler feed water into the drum. A constant heat supply is provided to the boiler, water vapours exit from the 'riser' pipes from the top of the boiler whereas liquid is drained downward from the bottom pipes [3].

Keeping the vapour pressure constant if the feed water is significantly colder than the temperature of steam in the drum, then increasing the flow of feed water causes a temperature drop which decreases the volume of the entrained vapour bubbles, following first-order behaviour and liquid level decreases initially [4]. With constant heat supply, the liquid level will start increasing in an integral form leading to a pure capacitive system.

The two systems may be modelled as

$$Y_1 = K_1/s$$
$$Y_2 = K_2/\tau*s + 1$$

Where Y_1 and Y_2 are the outputs; τ = time constant of first order process; K_1 = gain of pure capacitive process and K_2 = gain of first order process

The result of two opposing effects is given by:-

$$Y = Y_1 - Y_2 \Rightarrow Y = K_1/s - K_2/(\tau*s + 1)$$
$$\Rightarrow Y = ((K_1*\tau - K_2)s + K_1)/s(\tau*s + 1) \quad (1)$$

The transfer function has a positive zero at the point

$$s = -K_1/(K_1*\tau - K_2) > 0$$

The system shows inverse response only if it has a zero in the right half plane.

This condition is satisfied only when

$$K_1*\tau < K_2$$

Conversely on decreasing the flow of feed water, the volume of liquid increases abruptly for a short time. This happens because the volume of bubbles increases. This leads to first order behaviour. On the other hand, the steam supply remains constant and consequently the liquid level of the boiling water decreases in a pure capacitive response [5-7].

2. PROBLEM DEFINITION

The problem of inverse response has been an evergreen challenge for control engineers. So if the parameters are estimated then a suitable compensator for tackling this phenomenon may be properly designed. We are required to estimate the parameters for an inverse response of a system made up of a purely integrative process and a first order processes that act in opposite directions.

3. METHOD

This section helps in evaluating the process parameters by studying the experimental or simulated graph mathematically. For solving the equations we consider no dead time delay. From the graph shown in figure 1 we can see that its minima occurs at time ($t = t_1$). The function holds a zero value at time ($t = t_2$) and from observation it is clearly

^{1,2} Department of Electrical & Instrumentation Engineering, Thapar University, Patiala, India, ¹E-mail: mandy_tiet@yahoo.com, ²dhruvsaksena.tu@gmail.com

understood that the plot has its derivative as zero at $(t = t_1)$. Since the function remains purely integrative after a long time *i.e.* the effect of first order system is negligible so its derivative tends to a constant value say at infinite time.

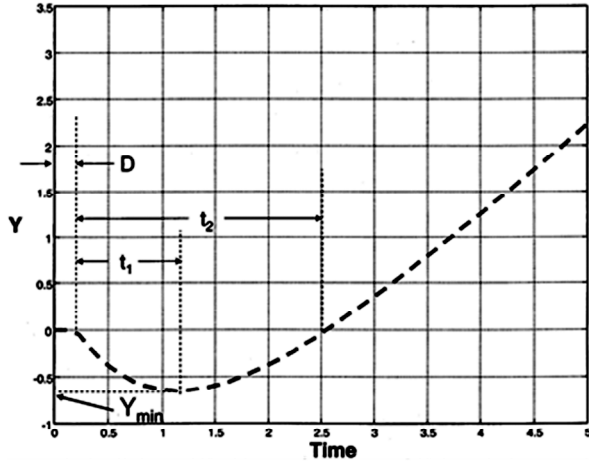


Figure 1: Inverse Response of First Order and a Pure Capacitive System

Here $Y(t)$ = value of function at time t ; D = dead time delay; t_1 = time to reach minimum value t_2 = value of time for which function holds zero value; Y_{\min} = minimum value of function

In figure 1, assume $D = 0$ for convenience, the various mathematical equations are:

$$\begin{aligned} Y(t) &= K_1 * t - K_2 (1 - \exp(-t / \tau)) \\ \Rightarrow Y(t) &= K_1 * t - K_2 + K_2 * \exp(-t / \tau) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{dy}{dt} &= K_1 - (K_2 / \tau) * \exp(-t / \tau) \\ \Rightarrow \text{At } t = \infty \lim_{t \rightarrow \infty} \frac{dy}{dt} &= k_1 \end{aligned} \quad (3)$$

$$\text{At } t = t_1 \quad dy/dt = K_1 - (K_2 / \tau) * \exp(-t_1 / \tau)$$

Since the derivative is zero at $t = t_1$, so the above equation can be written as:

$$K_1 - (K_2 / \tau) * \exp(-t_1 / \tau) = 0 \quad (4)$$

$$\text{At } t = t_2, Y(t_2) = 0.$$

$$\text{Therefore } Y(t_2) = K_1 * t_2 - K_2 + K_2 * \exp(-t_2 / \tau)$$

$$\Rightarrow K_1 * t_2 - K_2 + K_2 * \exp(-t_2 / \tau) = 0 \quad (5)$$

The function holds its minimum value at time (t_1) . Let it be called Y_{\min}

$$Y(t_1) = Y_{\min} \Rightarrow Y(t_1) = K_1 * t_1 - K_2 + K_2 * \exp(-t_1 / \tau)$$

$$\Rightarrow K_1 * t_1 - K_2 + K_2 * \exp(-t_1 / \tau) = Y_{\min} \quad (6)$$

Solving equation (4) and equation (6), we get

$$K_1 * (\tau) + K_1 * t - K_2 = Y_{\min} \Rightarrow K_1 * (\tau) + K_1 * t - Y_{\min} = K_2 \quad (7)$$

If we take any example we can see that we directly get the value of K_1 by using equation-(3). The value of Y_{\min} is available to us from the plot and we are left with two parameters which are τ and K_2 . To solve these two parameters we consider two equations (4) and (5).

Now substituting the value of K_2 from equation (7) to equation (5), the final result is stated below:

$$K_1 * t_2 - ((K_1 * t_1 + K_1 * \tau) - Y_{\min}) + ((K_1 * t_1 + K_1 * \tau) - Y_{\min}) * \exp(-t_2 / \tau) = 0$$

Since K_1, t_2, Y_{\min} are available to us, so we can get the value of τ by using the given matlab commands: $x = \text{solve}('K_1 * t_2 - ((K_1 * t_1 + K_1 * \tau) - Y_{\min}) + ((K_1 * t_1 + K_1 * \tau) - Y_{\min}) * \exp(-t_2 / \tau) x = \text{vpa}(x)$

In the above equations τ is replaced x and K_1, t_2, t_1, Y_{\min} are replaced by their corresponding numerical values. The solution gives a value of τ . This value is then put in the equation (7) to get the value of K_2 .

4. RESULTS AND DISCUSSIONS

This method of parametric estimation of Inverse Response is primarily dependent on the unit step of the process. For this purpose, we have to simulate this response using MATLAB SIMULINK. This is done by primarily following the equation (1) along with the constraint $K_1 * \tau < K_2$. The values of K_1, K_2 and τ are chosen for six different cases and the transfer function so formed is shown in table 1. From the response in the form of the simulated graph, the values of t_2, t_1 and Y_{\min} are obtained and the proposed method is applied for. The resulting value of estimated K_1, K_2 and τ are also shown in table 1. It is clearly seen that there is no error in estimation of K_1 , while the errors in K_2 and τ are only on account of the manual measurements done for of t_2, t_1 and Y_{\min} . The simulated graphs are shown in figure 2.

Table 1
Results of Parametric Estimation

S No.	Overall transfer function	K_1			K_2			τ		
		Actual value	Estimated value	Error %	Actual value	Estimated value	Error %	Actual value	Estimated value	Error %
a)	$14/(5s + 1) - 2/s$	2	2	0	14	14.28	1.85	5	5.109	2.18
b)	$30/4s + 1 - 5/s$	5	5	0	30	30.84	2.801	4	4.18	4.5
c)	$35/(7s + 1) - 3/s$	3	3	0	35	34.60	-1.01	7	6.854	2.08
d)	$6/s - 20/(2s + 1)$	6	6	0	20	20.57	2.887	2	2.104	5.4
e)	$3/s - 25/(6s + 1)$	3	3	0	25	24.72	-.892	6	5.903	-1.61
f)	$2/s - 13/(4s + 1)$	2	2	0	13	12.83	-1.25	4	3.911	-2.22

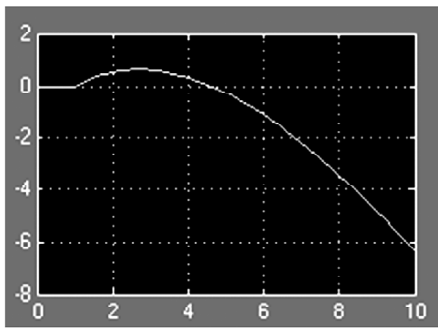


Figure 2a

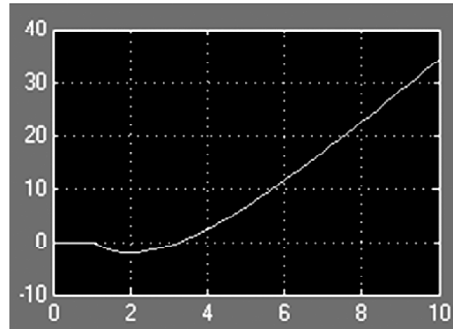


Figure 2d

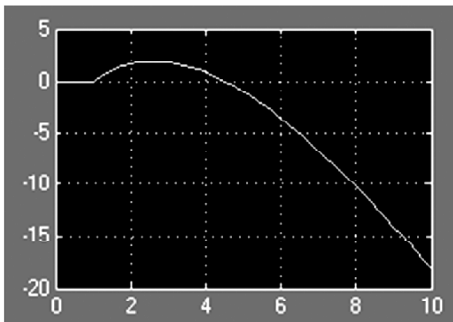


Figure 2b

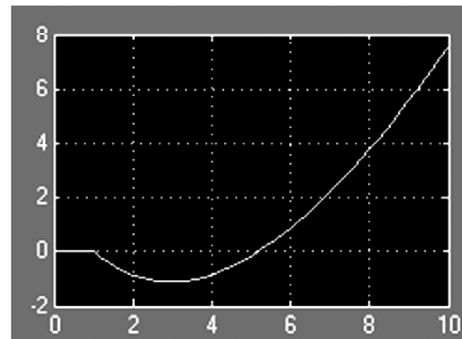


Figure 2e

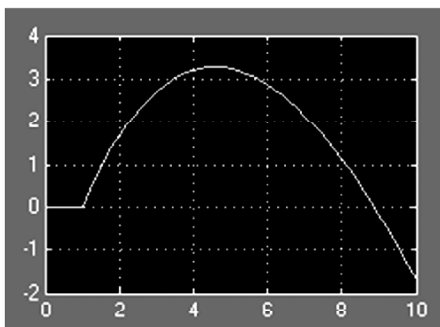


Figure 2c

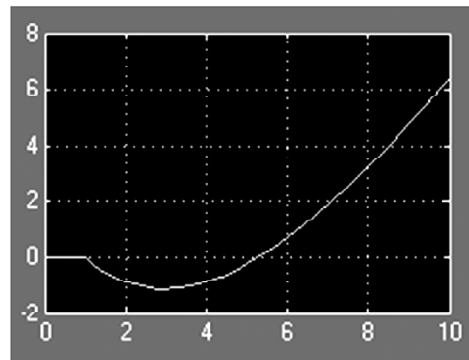


Figure 2f

5. CONCLUSION

In this paper, a novel method is presented to estimate the inverse response process parameters. Here opposing cases of a purely integrating process and first order process are taken. Further, the process is assumed to be without any time delay. The simulated graph is used for getting the inputs for solving a set of non-linear equations. The parameters are estimated within the acceptable level of errors.

6. FUTURE SCOPE

This method is easy to use and gives reliable process parameters. The proposed model and estimating procedure is suitable for application in industrial environment because of its simplicity. The results show that it can be applied in the estimation of large number of processes giving better estimation in less time. The parameters so estimated can be used for designing a suitable compensator so as to deal with the inverse response problem.

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