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MATHEMATICAL APPROACH OF DECISION SUPPORT SYSTEM FOR CUSTOMER-BASED PROBLEM

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ABSTRACT: A group of customers wait to receive service in the form of queue (waiting line). The formation of waiting line is a common phenomenon which occurs whenever the current demand for a service exceeds the current capacity to provide that service. The queue of peoples may be seen at cinema ticket window, reservation office, super market etc.

The customer arriving for service may form one queue and be serviced through only one station as in doctor's clinic or serviced through several stations as in barber shop. In this queue system customer arrivals form a Poisson process with constant arrival rate. If the customer's arrival rate is not constant but it depends on the number of customers present in the system then whole characteristic of this model will change. This new queuing theory for customer-based1 problem will discuss in this paper.

Keywords: Customer, Queuing Theory, Arrival and Service rate.

INTRODUCTION

From the time of birth until death human beings often find themselves waiting for things, events condition etc. A major topic of Applied Mathematics that deals with this phenomenon of waiting is called Queuing Theory. Using the word *Queue*, which is more common in British than American English and means a *line up* or *to form a line*, a closely reasoned body of mathematical theory has been developed to describe this common human activity.

Queuing Theory[2] arises from the use of powerful mathematical analysis to theoretically describe processes along with statistical probabilistic techniques to account for varying dynamic patterns within the stages of a productive process. In this chapter we describe the developments in new queuing methodology. We are primarily concerned with basic features such as general waiting time, waiting length and total customers in the system for different model. Since these features have eluded exact analysis, approximation procedures have been proposed. Proposed new queuing models have been applied in many domains including, Mall shop, railway reservation system, machine-failure problem, communication engineering and manufacturing System.

New queuing theory

- Model Description

It is single-server queuing model which consists of a single queue and a single service facility. Customer arrivals form a Poisson process with rate λ. equivalently, the customer inter arrival time are exponentially distributed with mean $1/\lambda$. The customer's arrival rate and service rate are not constant but arrival rate depends on the number of customers present in the system and service rate depends on nature of server. Assume that customers are served in their order of arrival (FCFS scheduling). This type of $M|M|1$ queuing model[3] is shown in figure 1.

- **Notations used**

Here first M stands for Poisson arrival (exponential inter-arrival time)

Second M stands for Poisson departure (exponential service time)

1 stands for single server

∞ stands for infinite capacity of the system

FCFS notation representing first come first served (service discipline)

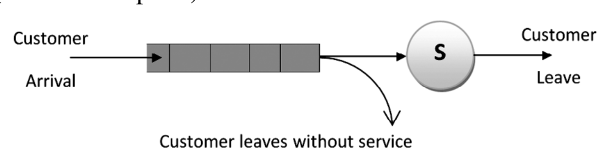

Figure 1: Customer Leave the Queue

- **Mathematical Description:**

Let us consider λ_n = mean arrival rate of customers when there are

N units present are the system.

μ = mean service rate

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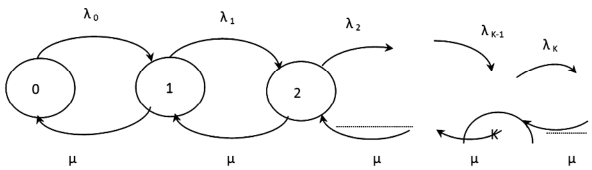

Figure 2: State Transition Diagram

From the figure 2 : The equations of balance of all state are given as follow

State zero : $\mu P_1 = \lambda_0 P_0$

$$P_1 = \frac{\lambda_0}{\mu} P_0$$

State one : $\lambda_0 P_0 + \mu P_2 = (\lambda_1 + \mu) P_1$

$$P_2 = -\frac{\lambda_0}{\mu} P_0 + \left(\frac{\lambda_1}{\mu} + 1\right) P_1$$

$$P_2 = \frac{\lambda_0 \lambda_1}{\mu^2} P_0$$

State two : $\lambda_1 P_1 + \mu P_3 = (\lambda_2 + \mu) P_2$

Proceeding in this way, we have

$$P_k = \frac{\lambda_0 \lambda_1 \lambda_2 \ldots \lambda_{k-1}}{\mu^k} P_0$$

But $\sum_{k=0}^{\infty} P_k = 1$

$P_0 + P_1 + P_2 + \ldots\ldots\ldots\ldots\ldots\ldots = 1$

$$\left[1 + \frac{\lambda_0}{\mu} + \frac{\lambda_0 \lambda_1}{\mu^2} + \ldots\ldots\ldots\right] P_0 = 1$$

$$P_0 = \frac{1}{S}$$

Where $S = 1 + \frac{\lambda_0}{\mu} + \frac{\lambda_0 \lambda_1}{\mu^2} + \ldots\ldots\ldots\ldots \infty$

We assume that S is convergent series because if S is a divergent series then $P_0 = 0$ and it is meaningless.

Let us consider $\ddot{e}_n = \frac{\lambda}{n+1}$. In this case $\ddot{e}_n$ decreases as n increases. That is arrival rate decreases with the increase in queue length.

Substitute the values of $\ddot{e}_n (n = 0, 1, 2, \ldots\ldots\ldots)$ we have

$$S = 1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{1.2.\mu^2} + \frac{\lambda^3}{1.2.3.\mu^3} + \ldots\ldots\ldots\ldots$$

Put $ó = \frac{\lambda}{\mu}$ in above equation. We get

$$S = 1 + \sigma + \frac{\sigma^2}{2!} + \frac{\sigma^3}{3!} + \ldots\ldots\ldots$$

$S = e^{\sigma}$ which is always convergent for σ to be finite.

Since $P_0 = \frac{1}{S}$

So $P_0 = e^{-\sigma}$ and

$$P_1 = \frac{\lambda_0}{\mu} P_0$$

$P_1 = \sigma e^{-\sigma}$ Proceeding in this way, we get

$$P_k = \frac{\sigma^n}{n!} e^{-\sigma}$$

Which implies that P_n follows the Poisson distribution.

- **Performance Measure**

➢ **Utilization**: The utilization gives the fraction of time that the server is busy.

Utilization = $1 - P_0$

Utilization = $1 - e^{-ó}$

➢ **Steady State Probability**: The steady-state probabilities[5] are given by

$$P_n = \frac{\sigma^n}{n!} e^{-\sigma}$$

➢ **Average number of customers in the system:** The average number[4] of customers in the system is given by:

$$E[N] = \sum_{K=0}^{\infty} K \,.\, P_k$$

$$E[N] = \left(\sum_{K=0}^{\infty} K \,.\, \frac{\sigma^k}{k!} e^{-\sigma}\right) \text{ where } \sigma = \frac{\lambda}{\mu}$$

$$E[N] = ó$$

The average number of customers in the system for very utilizations is shown in figure 4.11.

➢ **Average number of customer in waiting line** : Average number of customer in waiting line is given by:

$$E[W] = \sum_{K=0}^{\infty} (K-1) P_K$$

CONCLUSION

Queuing Theory arises from the use of powerful mathematical analysis to theoretically describe processes along with statistical probabilistic techniques to account for varying dynamic patterns which deals phenomenon of waiting line (Queue). A group of customers wait to receive service in the form of queue (waiting line). The formation of waiting line is a common phenomenon which occurs whenever the current demand for a service exceeds the current capacity to provide that service. Proposed queuing model has been applied in many domains like Mall shop, railway reservation system, machine-failure problem etc.

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