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# Wiener Filter Using Digital Image Restoration

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***Abstract:*** In this paper, restoration of images distorted by systems with noisy point spread functions and additive detection noise has been considered. The Wiener filter is one of the well established linear filtering methods and is widely known for its excellent performance in denoising the white noise. This filter is solution to the restoration problem based upon the hypothesized use of a liner filter and minimum mean square error (MMSE) criterion. Image filtering algorithms are applied on image to remove noise that is either present in the image during capturing or injected into the image during transmission. Performance of Wiener filter in frequency domain for image restoration is compared with that in the space domain on images degraded by white noise. It is also observed that the wiener filter having better performance for images corrupted by white noise compared to other non- linear filters.

***Keywords:*** Digital image, Fourier transform, Average, Wiener filter, PSNR.

## 1. INTRODUCTION

Restoration of images distorted by a system with a random blur function has been studied in recent years. The phenomenon of random blur describes an important class of imaging systems- those in which the impulse response function (or the transfer function) is random [1]. The uncertainty of random blur may result from random amplitude and phase fluctuations of the pupil function of the optical system, due to such effects as optical propagation through turbulence of dust particles on the lens. Random blur may also result from random vibrations of the imaging system relative to the object. Gaussian noise is an idealized form of white noise, which is caused by random fluctuations in the signal as shown in Figure 2.

The rest of paper is organized as follows. Section 2 describes the definition of digital image and pixel is shown. It demonstrates original image and noised image. In section 3 discuss the different smoothing techniques. Section 4 & 5 demonstrates the results conducted and finally this paper is concluded in section 6.

## 2. DIGITAL IMAGE DEFINITION

A digital image a[m, n] described in a 2D described space is derived from an analog image $d(i, j)$ in a 2D continuous space through a sampling process that is frequently referred to as digitization [1]. The effect of digitization is shown in Figure 1.

The 2D continuous image $d(i, j)$ is divided into N rows and M columns. The intersection of a row and a column is termed a Pixel. The value assigned to the integer to the integer coordinates [m, n] with m= {0,1,2,….M-1} and n = {0,1 ,2,….N-1} is a[m, n].

We will consider the case of 2D, monochromatic, static images [1, 2]. We have $d(i, j)$ and $n(i, j)$ represent the original image and additive noise, respectively. The observed degraded image $x(i, j)$ is given by

$$x(i, j) = d(i, j) + n(i, j) \quad (1)$$

The goal is to obtain a restored image $y(i, j)$ from $x(i, j)$, in which $y(i, j)$ should be equivalent to the original image $d(i, j)$ ideally.

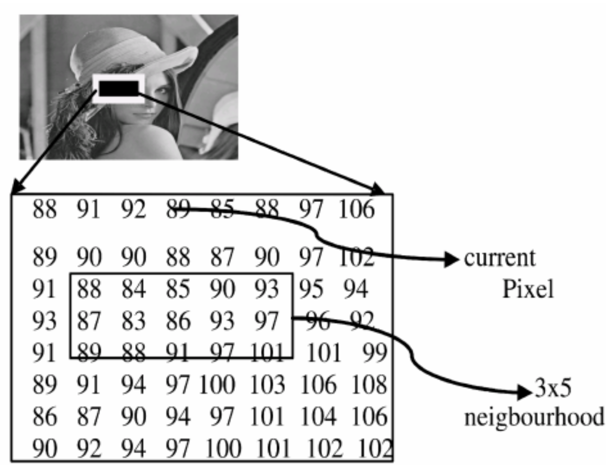


**Figure 1: (512 x 512) 'Lenna' Digital Image**

The value assigned to every pixel is the average brightness in the pixel rounded to the nearest integer value. The process of representing the amplitude of the 2D signal at a given coordinate as an integer value with L different gray levels is usually referred to as amplitude quantization or simply quantization [6].


**Figure 2: Image with White Noise**

## 3. SUMMARY OF SMOOTHING ALGORITHMS

Denoising techniques have received a great deal of attention in the field of image processing. A variety of smoothing filters have been developed which include linear filters, nonlinear filters and Fourier / wavelet transforms [3]. Image denoising problem is equivalent to that of image restoration when blurs are not included in the noisy image; their properties and domain of application have been studied extensively. We have:

### 3.1. Median filter

The Median Filter is performed by taking the magnitude of all of the vectors within a mask and sorted according to the magnitudes. It is based upon moving a window over an image (as in a convolution) and computing the output pixel as the median value of the brightness within the input window. The Simple Median Filter has an advantage over the Mean filter since median of the data is taken instead of the mean of an image. The pixel with the median magnitude is then used to replace the pixel studied. The median of a set is more robust with respect to the presence of noise. The median filter is given by

$$\left[ Filter(X_1 .......... X_N) = Median(\| X_1 \|^2 ...... \| X_N \|^2 \right]) \quad (2)$$

### 3.2. Average/mean Filter

The idea of mean filtering is simply to replace each pixel value in an image with the mean ('average') value of its neighbours, including itself. This has the effect of eliminating pixel values which are unrepresentative of their surroundings. Mean filtering is usually thought of as a convolution filter. Like other convolutions it is based around a kernel, which represents the shape and size of the neighbourhood to be sampled when calculating the mean. Often a 3×3 square kernel is used, although larger kernels (e.g. 5×5 squares) can be used for more severe smoothing. (Note that a small kernel can be applied more than once in order to produce a similar but not identical effect as a single pass with a large kernel.) [2].

### 3.3. Two Types of Wiener Filtering and it's Performance

We describe the principle of 2-D Wiener filters in the frequency domain and in the space domain for the purpose of restoration of an image degraded by white noise [8,9].

#### *3.3.1. Wiener Filter in the Space Domain*

If the output of the Wiener Filter is $y(i, j)$, it is represented by

$$y(i,j) = \sum_{m=-N}^{N} \sum_{n=-N}^{N} w(m,n)x(i+m, j+n) \quad (3)$$

the weights of the Wiener filter, $w(m, n)$, can be found by minimizing.

$$j = E(\{d(i,j) - y(i,j)\}^2) \quad (4)$$

Where is $E$ denotes expectation. The solution for $w(m, n)$ is obtained in a vector form as

$$w = R^{-1} p \quad (5)$$

Where

$$w = [w(-N,-N).......w(-N,N), w(-N+1,-N).......$$
$$.................w(-N+1,N)........w(0,0)......w(N,N)]^T$$

$$p = [p(-N,-N).......p(-N,N), p(-N+1,-N).......$$
$$.................p(-N+1,N)........p(0,0)......p(N,N)]^T$$

$$R = \begin{bmatrix} R(0,0)................R(0,2N).............R(1,0)...................R(2N,2N) \\ R(0,-2N).............R(0,0)................R(1,-2N)..............R(2N,0) \\ R(-1,0)...............R(-1,2N)..........R(0,0)...................R(2N-1,2N) \\ R(-2N,-2N)......R(-2N,0).........R(-2N+1),-2N)......R(0,0) \end{bmatrix}$$

These equations are known as the Wiener–Hopf equations. The matrix $R$ appearing in the equation is a symmetric Toeplitz matrix [5]. These matrices are known to be positive definite and therefore non-singular yielding a unique solution to the determination of the Wiener filter coefficient vector $w(m, n)$.

So $R(m, n)$ and $p(m, n)$ correspond to the autocorrelation function of $x(i, j)$ and cross – correlation function of $d(i, j)$ and $x(i, j)$, respectively, which are given by

$$R(m,n) = E[d(i,j)x(i-m, j-n)] \quad (6)$$

$$p(m,n) = E[d(i,j)x(i-m, j-n)] \quad (7)$$

Respectively. $\sum_{I=0}^{M-1} \sum_{J=0}^{M-1} \{d(i,j) - y(i,j)\}^2$ However, in practice, the following criterion

$$J = \frac{1}{M^2} \quad (8)$$

is often defined instead of $eq^n$ (4) due to its easy computation, and the solution in $eq^n$ (5) is obtained in this case $R(m, n)$ and $p(m,n)$ are calculated as

$$R(m,n) = \frac{1}{M^2} \sum_{I=0}^{M-1} \sum_{J=0}^{M-1} \{x(i,j) - x(i-m, j-n)\} \quad (9)$$

$$p(m,n) = \frac{1}{M^2} \sum_{I=0}^{M-1} \sum_{J=0}^{M-1} \{d(i,j) - x(i-m, j-n)\} \quad (10)$$

Respectively, from $M$ x $M$ images of $d(i,j)$ and $x(i,j)$

The output of the wiener filter is obtained by eq$^n$ (3) with the solution vector in eq$^n$ (5).

### *3.3.2. Wiener Filter in the Frequency Domain*

The Wiener filter could be implemented in the frequency domain by transforming the degraded image into the frequency domain using Discrete Fourier Transform (DFT), filtering and inverse transforming. In the frequency domain, the wiener filter is given by [7].

$$H(u,v) = \frac{P_D(u,v)}{P_D(u,v) + P_N(u,v)} \quad (11)$$

where $P_D(u,v)$ and $P_N(u,v)$ represent the power spectra of $d(i,j)$ and $n(i,j)$, respectively. This solution is derived in a similar way with that in the space domain. By minimizing

$$J = E[|D(u,v) - H(u,v)X(u,v)|^2] \quad (12)$$

Where $D(u,v)$ and $X(u,v)$ represent the discrete Fourier transforms (DFTs) of $d(i,j)$ and $x(i,j)$, respectively, the solution is first obtained as

$$H(u,v) = \frac{E[X(u,v)D^*(u,v)]}{E[|X(u,v)|^2]} \quad (13)$$

Where * denotes complex conjugate. When $n(i,j)$ is white noise, the numerator reduces to

$$\begin{aligned} E[X(u,v)D^*(u,v)] &= E[(D(u,v) + N(u.v)xD^*(u,v)] \\ &= E[|D(u,v)|^2] \\ &= P_D(u,v) \end{aligned} \quad (14)$$

And the denominator reduces to

$$E[|X(u,v)|^2] = P_D(u,v) + P_N(u,v) \quad (15)$$

Where $P_D(u,v)$ and $P_N(u,v)$ correspond to the power spectra of $d(i,j)$ and $n(i,j)$, respectively. Thus eq$^n$(13) results in eq$^n$(11).

The output of the wiener filter is given by

$$Y(u,v) = H(u,v)X(u,v) \quad (16)$$

$$y(i,t) = IDFT(Y(u,v)). \quad (17)$$

Where IDFT in eq$^n$(17) means the Inverse DFT.

## 4. RESTORATION RESULTS IN IDEAL CASE

We investigated the performance of different filters with wiener filter and than two types of wiener filter in an ideal case where original and noise images are known a priori. We used 10 images in SIDBA, each of which has a size of 512 x 512 gray scales. A white noise was generated and added to each images, resulting in the preparation of noisy images for each image [3.] The results obtained by each implementation of the two wiener filter on the degraded images are shown in figure (4). The window size of wiener filter in the space domain was set to 5 x 5. The SNR improvement in dB is defined as

*SNR* Improvement [dB] =

$$SNR \text{ Improvement [dB]} = 10\log_{10} \frac{NMSE[d(i,j), x(i,j)]}{NMSE[(d(i,j), y(i,j)]} \quad (18)$$

Where

$$NMSE[d(i,j), x(i,j)] = 100x \frac{\text{var}[(d(i,j) - x(i,j)]}{\text{var}[d(i,j)]} \quad (19)$$

$$NMSE[d(i,j), y(i,j)] = 100x \frac{\text{var}[(d(i,j) - y(i,j)]}{\text{var}[d(i,j)]} \quad (20)$$

The phrase peak signal-to-noise ratio is ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. It has a lower PSNR (a higher PSNR would normally indicate that the reconstruction is of higher quality) [4].

## 5. RESTORATION COMPARISON

(a)Average filter (b) Median filter (c)Wiener (in space domain) (d) Wiener filter (in frequency domain)

**Figure 4: Restoration Comparison of 'Lenna' Image (SNR = 0[dB])**

### 5.1. Simulation Results

**Table 1**
**Restoration Comparisons**

| *Filter's name* | *Mean* | *Std* | *MSE* | *PSNR* |
|---|---|---|---|---|
| Average | 124.1375 | 45.7045 | 0.0031576 | 74.1919 |
| Median | 124.482 | 45.1841 | 0.002914 | 74.748 |
| Wiener (space domain) | 124.0136 | 42.5885 | 0.0069597 | 75.42 |
| Wiener (frequency domain) | 124.4123 | 42.7868 | 0.0067237 | 75.9093 |

Table 1 shows a performance comparison of the denoising methods (Average filter, Gaussian filter, Median filter, Wiener filter in space domain, Wiener filter in frequency domain) in PSNR. Wiener filter in frequency domain have the highest value of PSNR. It gives the best performance for all images and noise levels.

## 6. CONCLUSION

The root mean – square errors (RMS) associated with filters are shown in figure. For this specific comparison, the wiener filter generates a lower error than any of the other procedures that are examined here. An image is degraded by white noise; the wiener filter is more suitable for restoration than a variety of smoothing filters such as the Gaussian, median, mean. In an ideal case where both the original and noise images are known, it has been found that the Wiener filter in the frequency domain is more effective than that in the space domain.

## 7. SCOPE FOR FUTURE WORK

There are a couple of areas which we would like to improve on. One area is in improving the de-noising along the edges as the method we used did not perform so well along the edges. The future work of research would be to implement Wiener Filter in Wavelet Domain.

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