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Effect of Optical Nonlinearities in wave Equation in Kerr-Type Nonlinear Non-Dispersive Medium

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Abstract: Optical nonlinearities give rise to many ambiguous effects in optical fibers. These effects have many useful applications, especially for the implementation of all-optical functionalities in optical networks. Kerr type nonlinearity is generally used and accurate in simulating optical applications. In this paper, a detail derivation and numerical solution of Schrodinger wave equation in nonlinear median has been reviewed. Solitary wave in Kerr like non dispersive medium with nonlinearity is studied. Different nonlinearity and the effect of nonlinearity in optical fiber have been studied. Nonlinearity effect of Schrodinger equation in z-direction has been studied. Type of nonlinearity that is considered is in refractive index in Kerr like medium.

Index Terms: Kerr type media, solitary wave.

1. INTRODUCTION

Electromagnetic wave propagation in nonlinear medium has been given more attention in last few decades due to its important application in optical signal transmission. Nonlinearity effects arise as optical fiber data rates, transmission lengths, number of wavelengths, and optical power levels increased. In Kerr like nonlinear medium Hasegawa [5] has shown that electromagnetic solutions, whose waveforms do not varies over the distance of propagation. The governing equation of nonlinear Schrödinger equation (NLSE) which renders the solutions. Vast research has been done in this area and the application of Kerr-like nonlinearity [5]-[7].

In practical applications materials exhibits a refractive index which varies with the electric field raised to a power other than two. The actual dependence of the index on electric fields is related by the physical processes such as diffusion and recombination. The processes give rise to nonlinearities. This type of nonlinearity is due to the variation of refractive index with change in electric field. Other type nonlinearity due to the scattering phenomena. The nonlinearities by scattering phenomena include stimulated Brillion scattering (SBS), stimulated Raman scattering (SRS), four wave mixing (FWM), self-phase modulation (SPM), cross-phase modulation (XPM), and intermediation [1] [4].

In this paper, a nonlinear wave equation with Kerr nonlinearity in non dispersive medium is presented which can be used in transmission application to improve the performance of system.

In this section we are using the nonlinearity due to refractive index. The general nonlinear medium where the refraction index intensity-dependence in following form [2]

$$n = n_0 + n_2 |E|^{\delta} \qquad (1)$$

Here δ is a arbitrary positive quantity. When it is equal to two the medium becomes Kerr-like medium. n_0 Refractive index of fiber core. n_2 Nonlinear refractive index coefficient due to electric field variation.

The analytic solution of nonlinear Schrödinger wave equation (NLSE) shows the effect of nonlinearity on wave equation. This effect of nonlinearity is more pronounce in optical application.

2. GOVERNING EQUATIONS

Consider the electric field envelope $\varnothing(z, t)$ in a monomode fiber [5]. Neglect losses and dispersion. The electric field is then:

$$E(z, t) = \varnothing\,(z, t) \exp[\,j(w_0 t - \beta_0 z)] \qquad (2)$$

Here z is the distance in the transmission direction of fiber. Nonlinear refraction index is given in (1)

Schrödinger wave equation [3]

$$-\frac{h^2}{2m}\nabla^2\varphi + V\varphi = E\varphi \qquad (3)$$

Wave propagate in space can be describe by wave equation which can be drive from above wave equation

$$\beta_0 = \frac{n_0 w_0}{c} \qquad (4)$$

In equation (1) if δ equal to two the medium behave like Kerr medium and is given by as follows

$$n = n_0 + n_2\,|E|^2 \qquad (5)$$

3. APPROXIMATIONS

- The second derivative of can be neglected since they are much smaller than the terms $w_0^2\varnothing$ and $\beta_0^2\varnothing$ [1].
- n_2^2 Can be neglected since n_2 is very small [1].
- Only the leading term $-w_0^2\,|\varnothing|^2\,\varnothing$ of the second time derivative $\frac{\partial^2(|\varnothing|^2)}{\partial t^2}$ needs to be kept since the products of all additional terms with n_2 are negligibly small [1].

4. ANALYTICAL SOLUTION OF SE WITH REFRACTIVE INDEX NONLINEARITY

In this section we will find out the analytical solution of Schrodinger equation (SE) with refractive index nonlinearity.

To find the solution consider wave equation propagating in z-direction is given by [8]

$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 (n^2 E)}{\partial t^2} \qquad (6)$$

Using electric field and nonlinear refractive index given by equation (10) gives

$$\frac{\partial^2\{\varnothing \exp[j(w_0 t - \beta_0 z)]\}}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,[n = n_0 + n_2\,|E|]^2 \qquad (7)$$

Solving left hand side of this equation

$$\frac{\partial^2 E}{\partial z^2} = \begin{bmatrix} j^2\beta_0^2\varnothing \exp[j(w_0 t - \beta_0 z)] - 2j\beta_0 \exp \\ [j(w_0 t - \beta_0 z)]\dfrac{\partial\varnothing}{\partial z} \end{bmatrix} \qquad (8)$$

Solving right hand side of equation (7)

$$\frac{1}{c^2}\frac{\partial^2 (n^2 E)}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,\{[n_0^2 + n_2^2\,|E|^4 + 2n_0 n_2\,|E|^2]E\} \qquad (9)$$

Equation (2) becomes

$$|E|^2 = |\varnothing|^2$$

Using equation (2) and approximations

$$\frac{n_0^2}{c^2}\frac{\partial^2(E)}{\partial t^2} + \frac{2n_0 n_2}{c^2}\frac{\partial^2}{\partial t^2}(|E|^2\,E) = \frac{n_0^2}{c^2}\frac{\partial^2}{\partial t^2}\{\varnothing \exp[j(w_0 t - \beta_0 z)]\}$$
$$+\frac{2n_0 n_2}{c^2}\frac{\partial^2}{\partial t^2}[|\varnothing|^2\,\varnothing \exp[j(w_0 t - \beta_0 z)]] \qquad (10)$$

Using equation (8) and (10) we get

$$\frac{\partial\varnothing}{\partial z} + \beta_0^*\frac{\partial\varnothing}{\partial t} = -j\beta_0\frac{n_2}{n_0}|\varnothing|^2\,\varnothing \qquad (11)$$

Where

$$\frac{n_0}{c} = \beta_0^*\ \mathrm{s}$$

Above equation gives the nonlinear wave equations. Above nonlinear wave equation improve the data rate, transmission length and optical power level.

5. CONCLUSION

In this paper electromagnetic wave propagate in Kerr like medium is investigated and analytical solution of nonlinear Schrodinger wave equation is presented. Nonlinearity effect of refractive index is considered. Hereby briefly review the different kinds of optical nonlinearities encountered in fibers. The information is very useful in the design of the high bit rate optical devices.

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