

# Analytical BER Computation for MC CDMA System with Iterative HIC Receiver

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**Abstract:** *The existing mobile communication system suffer from problems like frequency selective fading and multi-path signals, multiple access interference due to imperfect non-orthogonal codes and narrow band interference due to coexisting systems. Multi-carrier Code Division Multiple Access (MC-CDMA) technique can combat these problems efficiently owing to its inherent properties. The objective of this paper is to suggest an optimum receiver for the MC-CDMA system by computing the BER of different receivers. Since multi-user interference degrades the performance of the receiver, interference cancellation technique is to be incorporated in the receiver. As Parallel Interference Cancellation (PIC) is faster, less efficient and Successive Interference Cancellation (SIC) scheme is slow but efficient a Hybrid Interference Cancellation (HIC) scheme is arrived at combining the advantages of these two schemes. The hybrid scheme is simple, faster and reliable. The performance of the receiver is studied by comparing with the BER of a detector without interference cancellation. Improvements in performance of parallel interference cancellation can be achieved by using multiple stages of the cancellation unit. Similarly performance of HIC receiver can also be improved if the PIC part of HIC is made iterative. Computation results imply iterative HIC outperforms iterative PIC compared in terms of computational complexity and error performance and hence will be an optimal choice for MC CDMA reception.*

**Keywords:** *MC CDMA, Iterative interference cancellation, SIC, PIC, HIC, BER.*

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## 1. INTRODUCTION

Cellular mobile communication is the mode of communication today, but will be the mode of living tomorrow. Future wireless services are intended to support multimedia services to millions of subscribers, which will be obviously leading to bandwidth shortage. Considering the state of fact, International Tele-communication Union has identified Code Division Multiple Access (CDMA) as the wireless access scheme. Multi Carrier CDMA (MC CDMA) will be the probable access scheme for beyond 3G services. The main objective of this paper is to analytically compute the BER of the MC-CDMA system with successive interference cancellation receiver, parallel interference cancellation receiver and hybrid interference cancellation receiver. BER is computed for iterative structures of parallel interference cancellation receiver and hybrid interference cancellation receiver. It is proposed to suggest an optimal version of reception scheme for MC-CDMA system based on the BER computation. Multi-carrier CDMA for multi-users, multi-service environment is exhaustively discussed in [1]. Multi-user detection for synchronous CDMA is analyzed by Verdu *et al* as early as 1989, followed by Varanasi *et al* for asynchronous CDMA system [2, 3]. In 1993 Patel *et al* analyzed the simple successive interference cancellation scheme for DS-CDMA system and in 1995 Divsalar *et al* analyzed the performance of DS-CDMA

system with parallel interference cancellation [4, 5]. In 1996 Pulin Patel *et al* compared the performances of DS-CDMA system using successive Interference Cancellation(IC) scheme and parallel IC scheme under fading [6].

## 2. MULTI CARRIER CDMA

CDMA is a multiplexing technique where many users at the same time asynchronously access the same frequency band by spreading their information with pre assigned unique code sequence. Recently CDMA technique is considered as a candidate to support multimedia services in mobile radio communication to provide higher capacity over TDMA and FDMA scheme. However, OFDM is also a promising choice in the field of radio communication as it can transmit high data rate in a mobile environment. In 1993 three types of new multiple access schemes such as multi carrier CDMA (MC-CDMA), multi carrier DS-CDMA (MC-DS CDMA) and multi tone CDMA (MT CDMA) were proposed based on combination of CDMA and OFDM [7]. Since FFT is incorporated, it is possible to realize transmitter and receiver without much complexities for the above three schemes. These schemes have high spectral efficiency due to minimally densely sub carrier spacing.

### 2.1 MC-CDMA Transmitter

The MC-CDMA transmitter spreads the original data scheme over different sub carriers using a given spreading code in the frequency domain. Normally orthogonal Walsh code is

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used in the down link so that attention is not necessary to the auto correlation characteristic of the spreading code. Fig.1 depicts the MC-CDMA transmitter of the  $k^{th}$  user, where  $N$  denotes the processing gain/the number of sub carriers and  $C_k(1) C_k(2) C_k(3) \dots C_k(N)$  is the spreading code of the  $k^{th}$  user. The number of sub carrier selection and the guard interval is optimally arrived based on BER minimization and to increase the robustness against frequency selective fading.

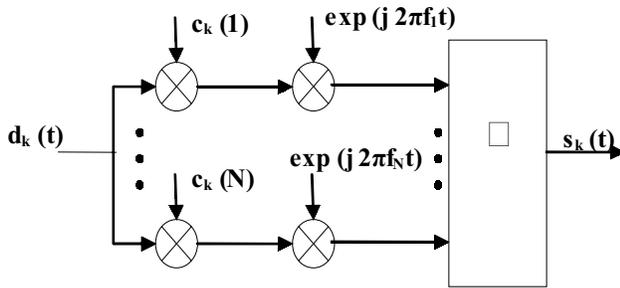


Figure 1: MC-CDMA Transmitter

2.2 MC-CDMA Receiver

A typical MC CDMA receiver is shown in Fig.2, where after the serial to parallel conversion, the  $N^{th}$  sub carrier is multiplied by the code  $C_k(N)$  to combine the received signal energy scattered in the frequency domain.

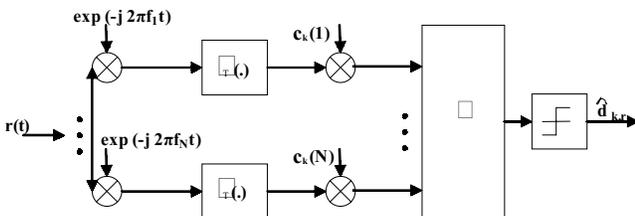


Figure 2: MC-CDMA Receiver

2.3 Multi-user CDMA Receivers

Multi-user receivers use knowledge of the spreading sequences to exploit the structure of Multiple Access Interference (MAI). The major difference between single-user detector and Multi-User Detector (MUD) is that in MUD, the users are jointly detected for their mutual benefit. Due to the lack of synchronization between users on the reverse link, MAI is correlated with the desired user's signal. Apart from this, due to near-far effect even small amount of correlation can potentially degrade the desired user's performance, since its interference is stronger. Multi user detection is a promising approach to overcome the limitations of single user CDMA receiver and improve system capacity.

2.3.1 Optimal Receiver

Verdu presented an optimum multi-user receiver, which requires a priori knowledge of the signal amplitudes and phases and involves a high degree of computational

complexity [8]. The optimum receiver is shown in Fig.3. The optimum receiver consists of a bank of  $k$  single user receiver whose outputs are then fed to a maximum likelihood viterbi decision algorithm. The optimum receiver requires a priori knowledge of the signal amplitudes and phases in order to derive a maximum decision statistic in the decoding. This decoder will introduce a considerable delay to achieve optimality and will have complexity in the order of  $2^k$  for every bit decision required. However, it was shown that the receiver is near-far resistant regardless of received power levels with significant improvement over the single user receiver. Because  $2^k$  computations are needed for every user's bit decisions, it should be obvious that for a high capacity system the receiver will not be capable of sustaining such a load. Hence optimal receiver has been used as a standard against which sub-optimal receiver's performance can be compared. Sub-optimal receivers extensively surveyed in literature are de-correlator, Minimum Mean Square Error (MMSE) receiver and linear multi-user detectors.

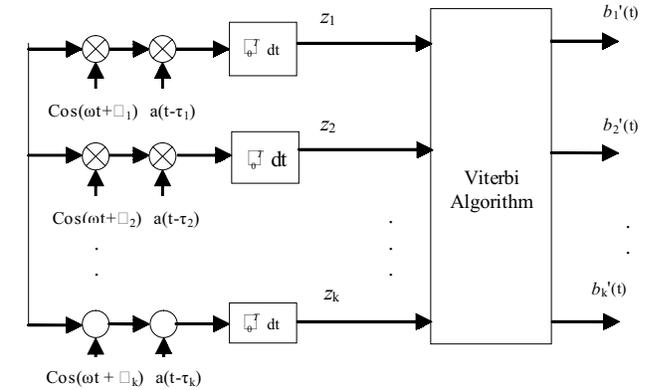


Figure 3: Optimal Receiver

2.3.2 The De-correlator

A linear detector's detection metric is given as

$$\hat{b} = \text{sgn}[\cos(\hat{\theta})T_{y_I} + \sin(\hat{\theta})T_{y_Q}] \tag{1}$$

If  $T = R^{-1}$ , linear detector is a de-correlator. As the name implies, the de-correlator removes the correlation between the elements of  $Y$ .

More explicitly,

$$\begin{aligned} \hat{b} &= \text{sgn}[\cos(\hat{\theta})R^{-1}Y_I + \sin(\hat{\theta})R^{-1}Y_Q] \\ &= \text{sgn}[Wb + R^{-1}n] \end{aligned} \tag{2}$$

where  $W$  is a  $KN_b \times KN_b$  diagonal matrix of the square root of user energies,  $b$  is a  $KN_b$  length vector with  $N$  element equal to the  $i^{th}$  data symbol of the  $k^{th}$  user. Assuming perfect phase estimates and delay estimates the probability of symbol error of the  $k^{th}$  user can be represented as

$$P_{k,i}(E) = Q \left( \sqrt{\frac{W_{j,j}}{(R^{-1})_{j,j} N_o}} \right) \tag{3}$$

where  $N_o$  is the one-sided noise power spectral density of AWGN.

Thus the performance of the de-correlator is identical to the single user case with the exception of the noise enhancement factor ( $R^{-1}$ )<sub>jj</sub>. The de-correlation receiver performs well at high SNR because MAI is eliminated by the inversion of the cross-correlations but, de-correlator fails since matrix inversion is computationally intensive and numerically sensitive. De-correlator also performs poorly at a lower range of SNR because the noise is multiplied by the inverse cross correlation matrix. Modification of optimum receiver leads to another class of sub-optimum receiver known as MMSE receiver.

### 2.3.3 Minimum Mean Square Error Receiver

In this case, linear transformation  $T=R^{-1}$  used in de-correlator is replaced by  $(T=R+N_o/2w^2)^{-1}$ . The performance of MMSE approaches that of de-correlator as  $N_o \rightarrow 0$ , but as  $N_o$  increases, performance reduces to that of conventional receiver. At low  $E_b/N_o$  MMSE receiver outperforms the de-correlator while at high  $E_b/N_o$ , de-correlator's performance approaches that of MMSE receiver. But, since computational complexity of MMSE is equal to that of de-correlator and its near-far resistance is slightly lesser than that of de-correlator, linear MUD like interference cancellation receivers are considered.

## 3. INTERFERENCE CANCELLATION SCHEME

In an interference cancellation scheme, the signal is first passed through a bank of correlators and then each user's signal is re-constructed and cancelled from the received signal. This process may also repeat for multiple stages. The interference cancellation techniques can be broadly broken into successive and parallel schemes for canceling multiple access interference.

### 3.1 Successive Interference Cancellation Scheme

Successive interference cancellation receiver is shown in Fig. 4. The detection is a successive process, with each user decoded in turn. After decoding  $k^{\text{th}}$  user, the estimated bits  $\hat{b}_k(t)$  are re-encoded to form an estimate of the received signal for that user,  $Z_k$ . This is then subtracted from the current composite signal  $Y_k$  to form a cleaner signal that can be used to find the bits for the user  $k+1$ .

The received signal is given by

$$r(t) = \sum_{n=-\infty}^{\infty} \sum_{k=1}^K \sqrt{P_k} \sum_{n=1}^M \beta_{k,m}(t) n b_k[n] u_T(t - nT - \xi_k) + \eta(t) \quad (4)$$

$$R\{C_{k,m}[n] e^{j(\omega_n t + \phi_{k,m}(t))}\}$$

where  $\xi_k$  is the relative time misalignment of the user  $k$  and uniformly distributed over the interval  $[0, T]$ ,  $\phi_{k,m}(t) = \varphi_{k,m}(t) + \theta_{k,m} - \omega_m \xi_k$  is the received phase and  $\eta(t)$  is Additive White Gaussian noise with two-sided PSD note. For each user  $k$ ,

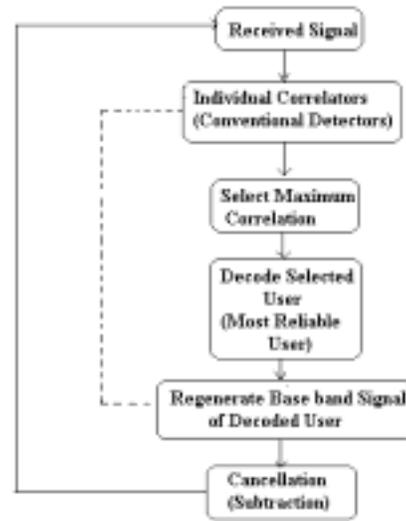


Figure 4: Successive Interference Cancellation Scheme

the composite signal used for detection is  $Y_k(t) = [y_{k,1}(t) y_{k,2}(t) \dots y_{k,m}(t)]^T$ . For the first user to be de-coded the composite signal is simply

$$Y_1(t) = r(t) \mathbf{1}_M \quad (5)$$

where  $\mathbf{1}_M$  is an  $M$ -vector of user. In general, the composite used signal for detection of user  $k$  is described by its sub-carriers.

$$y_{k,m}(t) = y_{k-1,m}(t) - Z_{k-1,m}(t), \quad k \geq 2 \text{ for all } m. \quad (6)$$

where  $Z_{k,m}(t)$  is an estimate of the received signal for user  $k$  on sub-carrier  $m$  and is

$$Z_{k,m} = \sqrt{P_k} R\{\hat{h}_{k,m} C_{k,m} b K e^{j(\omega_m(t-C_k) + \theta_{k,m})}\} \quad (7)$$

Hence, at each stage, the interference of the last decoded user is subtracted out of the signal so that the next user experiences less total interference. The received signal is processed successively, yielding the following decision static for user  $k$ ,

$$U_k = 1/T \int_0^T \sum_{m=1}^M y_{k,m} \{\hat{q}_{k,m} C_{k,m} e^{-j(\omega_m(t-\xi_k) + \theta_{k,m})}\} dt \quad (8)$$

where  $\hat{q}_{k,m}$  is a complex adjustment for amplitude and phase, dependent on the sub-carrier combination strategy.

The main problem with SIC is, since each user's signal must be estimated and subtracted out from the composite signal before decoding the next user, future users will not be decoded reliably if the present signal estimation is sufficiently inaccurate. Also SIC takes more time while reducing hardware. Apart from the above problems, a specific ordering of user powers must be enforced for the users to achieve similar performance. Finally, multi path propagation poses a particular problem as each multi path component must be cancelled. In fact, capacity in many interference cancellation systems drops off proportionally

to the number of multi path components. Indeed, the multi-path degradation can be effectively reduced by going for MC-CDMA scheme.

### 3.2 Parallel Interference Cancellation Scheme

In a PIC detector depicted in Fig.5, all the users are detected in parallel, at the same time. The initial estimates of the transmitted data of the  $n^{\text{th}}$  user are achieved by de spreading the received signal  $r(K)$  with the respective modified sequences  $Z_n^{(j,2)}(K)$  leading to

$$X_{n,0}^{(1,2)}(i) = \sum_{j=1}^{k-1-t_p} r(i-L+l) - Z_n^{(1,2)}(L-\tau_n) \quad (9)$$

The hard decision  $\text{sgn}(X_{n,0}^{(1,2)}(i))$  corresponds to the estimates  $l_n^{r(1,2)}(i)$  of a conventional receiver. For each user, the corresponding interference signal replica are subtracted from the receiver signal  $r(K)$ , generating interference mitigated signal. For the  $n^{\text{th}}$  user, these signals  $r_{n,L}^{(1,2)}(K)$  are generated according to

$$\begin{aligned} r_{n,1}^{\{1,2\}}(k) = & r(k) - \sum_{j=1}^{n-1} X_{j,0}^{(1,2)}(i) Z_j^{(1,2)}(K - iL - T_j) \\ & - \sum_{j=1}^{n-1} X_{j,0}^{(1,2)}(i+1) Z_j^{(1,2)}(K - (i+1)L - T_j) \\ & - \sum_{j=n+1}^m X_{j,0}^{(1,2)}(i-1) Z_j^{(1,2)}(K - (i-1)L - T_j) \\ & - \sum_{j=n+1}^N X_{j,0}^{(1,2)}(i) Z_j^{(1,2)}(K - iL - T_j) \end{aligned} \quad (10)$$

The main advantage of PIC is its fastness. But it suffers from the disadvantage that lower power users will have their BER very high, since detection is with less SIR. Hence, the performance of PIC is inferior to SIC.

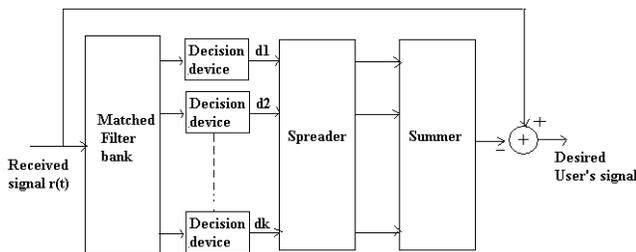


Figure 5: Parallel Interference Cancellation Scheme

### 3.3 Hybrid Interference Cancellation Scheme

Successive IC yields better performance with lot of processing time and parallel IC is superior to SIC in terms of time consumption but is inferior in terms of BER. Hence a mix of SIC and PIC will yield an optimal result. The main

idea behind hybrid IC is that instead of canceling all K users either in series or in parallel, they are cancelled partially in parallel and partially in series. The configuration for cancellation will be K-P-S, where K is the total number of users and the number cancelled in parallel and in series at each stage is denoted by P and S, respectively. The flow diagram of HIC is given in Fig. 6.

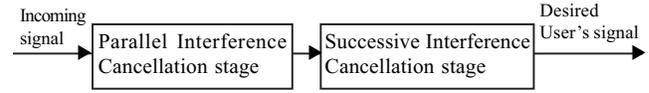


Figure 6: Hybrid Interference Cancellation

The signals of the first P stronger users (out of K) are chosen to perform PIC between them. As a result of this action, the P most reliable users are chosen, and their signals reconstructed in order to subtract them from the buffered version of the received signal. Now, 'P' signals are subtracted from the received signal. After that remaining K-P (i.e.S) users are arranged according to their strength and one by one, users are detected, subtracted and ultimately using SIC all the users are detected. Obviously, HIC performs in an optimal way when compared with SIC and PIC. Many researchers worked on optimizing the value of P and S, but in this work choosing of P and S is in an optimistic way. Based on the type of service offered target BER is decided, based on the modulation scheme used SNR yielding the target BER is decided. This SNR is used as a threshold to decide whether the user should be detected in PIC mode or SIC mode. i.e. those users having SNR greater than the threshold will be detected using PIC (P) since it will yield the performance required and the remaining users are detected through SIC means(S).

### 3.4 Iterative PIC

The performance of a single-stage PIC will be better if the data from all interfering users is known a priori. In reality, where such data is unknown, the PIC can be implemented in multiple stages. Specifically in the  $n^{\text{th}}$  stage of cancellation, the detector uses the bit decision in the  $(n-1)^{\text{th}}$  stage to regenerate the MAI and then subtracts it completely from the received signal of the desired user. As a result, when the estimated information from the previous stage becomes more accurate, the performance of multistage PIC will be better.

### 3.5 Iterative HIC

The performance of iterative PIC structure can still be improved by going for an iterative HIC structure. Improvements in HIC can be achieved by using more stages of the PIC but after certain stages of PIC, significant improvement in performance cannot be realized. Hence, an iterative HIC with two-stage PIC is modeled and analyzed.

## 4. ANALYTICAL MODELING AND RESULTS

The mathematical modeling is carried out in order to emulate the real time situation with a greater degree of precision. It

also allows us to explore the more subtle points which a simulation cannot bring out. In modeling the MC-CDMA system, 10 unequal power users are considered with a processing gain of 32. The number of bits processed to get the BER performance of the system is 1000. The channel is assumed to be multi path Rayleigh fading in AWGN floor.

Fig.7 brings out the comparison between the performance of CDMA and MC-CDMA systems. The MC-CDMA system shows an improved BER performance over the CDMA system. This is because of the frequency diversity present in MC-CDMA technique. The performance of the MC-CDMA system can be further improved by increasing the number of carriers.

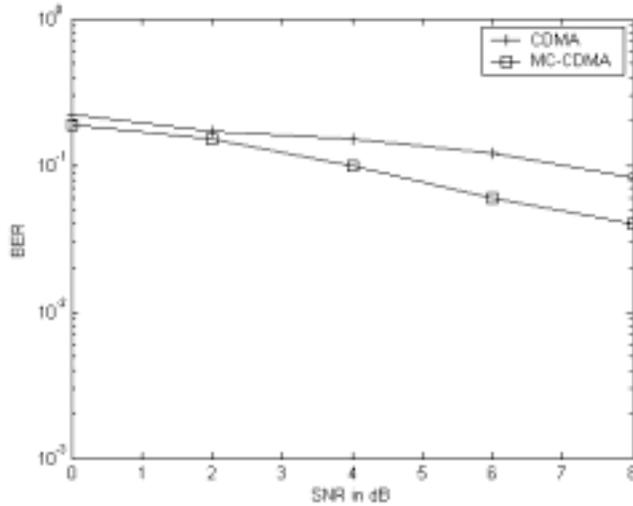


Figure 7: Comparison of CDMA and MC-CDMA System

Figs. 8 and 9 shows the BER performance of MC-CDMA system with SIC and PIC scheme. The SIC scheme improves the BER performance of the MC-CDMA system to a very good extent. The drawback of the SIC scheme is that it introduces delay in the functioning of the system. The PIC scheme improves the BER performance of the MC-

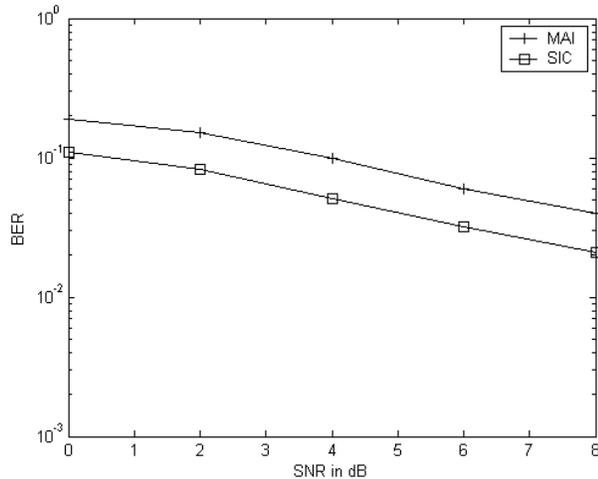


Figure 8: Performance of SIC Receiver

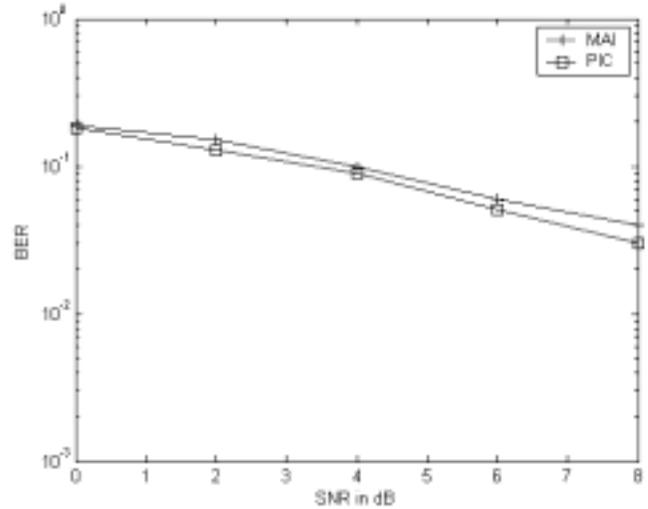


Figure 9: Performance of PIC Receiver

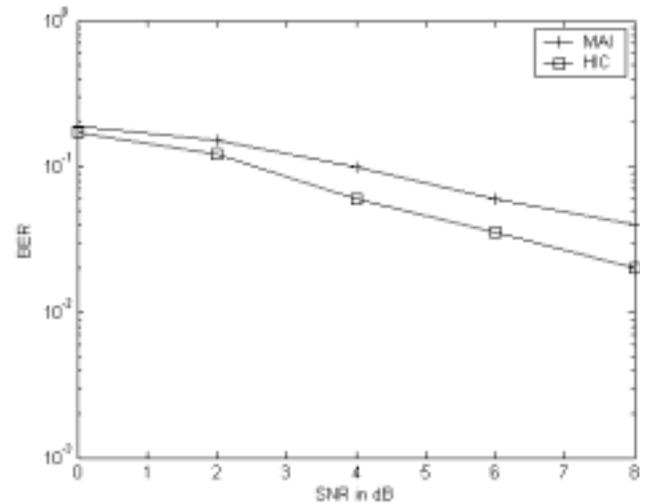


Figure 10: Performance of HIC Receiver

CDMA system but not to the extent of SIC scheme. Indeed PIC does not introduce much delay in the functioning of the system. Fig. 10 illustrates the BER performance of the MC-CDMA system with HIC scheme. The HIC scheme provides a BER performance which is better than the PIC scheme but not to the extent of SIC scheme. The HIC scheme is an optimal scheme which brings a good tradeoff between SIC and PIC scheme in terms of BER and delay. Fig. 11 brings out the comparison between different interference cancellation schemes. It is very evident that MC-CDMA system with SIC scheme provides the best BER performance. The BER performance of the rest of the systems is in the order of MC-CDMA system with HIC scheme, MC-CDMA system with PIC scheme and MC-CDMA system with no interference cancellation scheme.

Fig. 12 is the comparison between MC-CDMA system with PIC scheme and MC-CDMA system with iterative PIC scheme. The graph depicts the improvement provided by

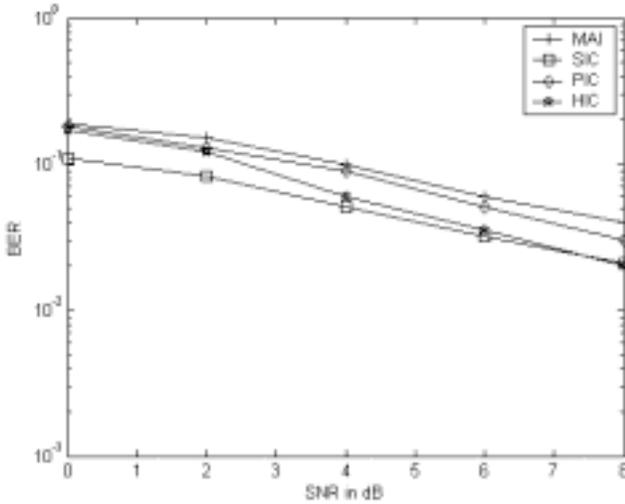


Figure 11: Comparison of SIC, PIC and HIC Receivers

and MC-CDMA system with iterative HIC scheme. The graph depicts the improvement provided by iterative HIC scheme over the HIC scheme. Fig. 14 brings out the comparison between the MC-CDMA system and MC-CDMA system with different interference cancellation schemes. The iterative HIC scheme provides the optimum performance than any other scheme.

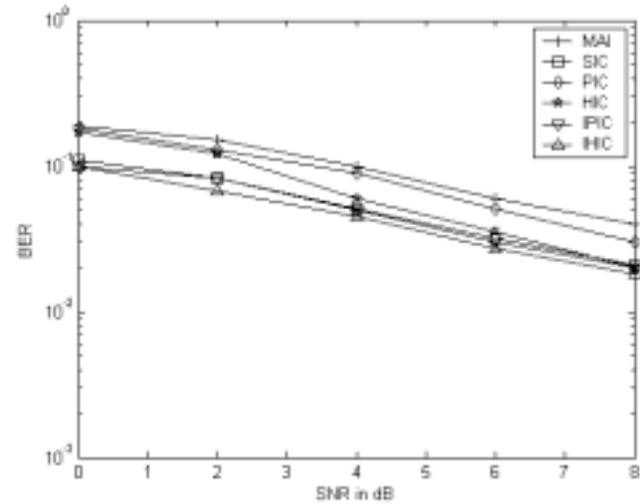


Figure 14: Performance of Interference Cancellation Receivers

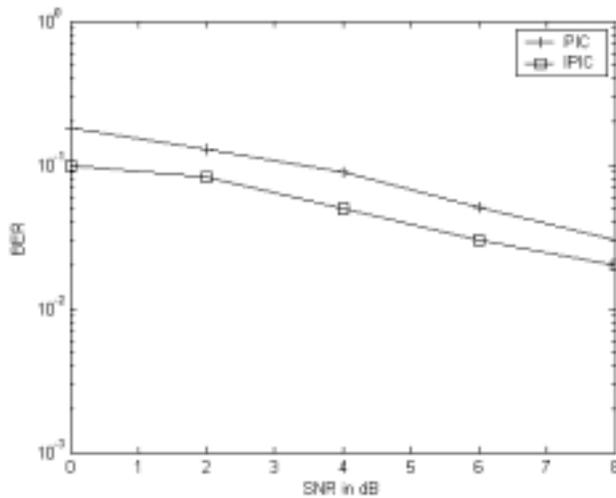


Figure 12: Performance of Iterative PIC Structures

iterative PIC scheme over the PIC scheme. Fig. 13 is the comparison between MC-CDMA system with HIC scheme

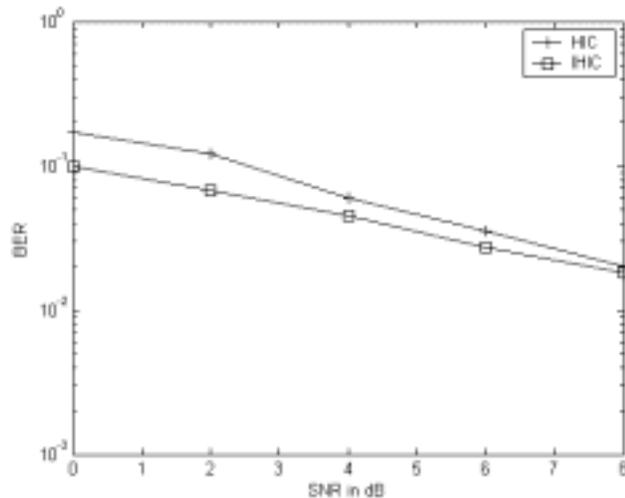


Figure 13: Performance of Iterative HIC Structures

### 5. CONCLUSION

In this paper, various types of sub-optimal multi-user receivers are analytically modeled and BER is computed. Results reveal MC-CDMA system with interference cancellation performs well when compared with MC-CDMA systems using single-user receiver. Considering all the interference cancellation receivers, performance of SIC is superior. But, since it consumes much time, time effective PIC is realized. The problem with PIC is its poor performance. So, a compromise among SIC and PIC is arrived. This hybrid interference cancellation receiver performs well when compared with PIC but is far inferior to SIC. Moreover, optimizing the threshold is a big issue in HIC. In this work, threshold in HIC is decided based on the target SNR, BER required for the service offered and the type of modulation used. To still improve the performance of hybrid interference cancellation receivers, iterative structures are formulated. Since, iterative SIC will not provide any improvement in performance, iterative PIC structure is analytically modeled. Performance of two-stage PIC is studied and is found to be better when compared with single stage PIC, but it is still not matching the performance of SIC. So, an iterative HIC is modeled and its performance is studied. Iterative HIC scheme with two stages of PIC followed by a single stage of SIC provides an error rate which is optimal considering both complexity and performance. Hence, it is concluded that the iterative HIC structure will be an optimal multi-user receiver structure for MC-CDMA system.

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