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Mitigation of noise effect with the support of LMS and RL

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Abstract-In most signal processing scenarios, the statistical characteristics of the information signal and the noise signal are not known to us beforehand so a fix coefficient filter can't be employed. Mostly, the signal and noise overlap in a common frequency range in such scenarios. This paper presents Adaptive Filters as a solution for noise cancellation. Various adaptive filtering algorithms such as LMS, RLS etc have been compared in terms of performance, convergence, complexity etc.

Keywords-LMS, RLS, NLMS, Kalman Filter, Adaptive Filters, Noise Cancellation

Introduction

Noise, or unwanted signals plague information signals in almost all domains of signal processing. Over the years, many techniques have been developed to counter or suppress these unwanted signals. The fundamental idea used to be to try to cancel out the noise signal by the use of a noise-cancelling speaker, for the case of speech signals, that emits signal with the same amplitude but with the opposite or inverted phase to the noise signal. Modern methods of noise cancellation started employing a somewhat different strategy. Now, the usually employed methods of estimating a signal affected by noise of an additive nature involve passing it through a filter that tries to repress the noise without affecting the information signal [1-5]. Earlier and more trivial implementations of such filters were fixed i.e. their coefficients were fixed. The design of filters having fixed coefficients requires beforehand knowledge about both the signal and the noise. More recent, and more advanced implementations of filters are adaptive i.e they have the ability to 'adapt' according to the incoming signal and hence adjust their parameters, or coefficients. The major advantage that adaptive filters offer is that their design requires no beforehand knowledge about the information signal or the noise characteristics [6-9].

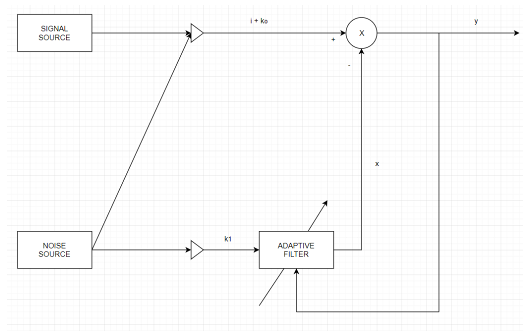


Figure 1. Basic Structure of Adaptive Filter

Figure 1 shows an oversimplified structure for the adaptive noise cancellation problem. A information signal *'i'*, transmitted through a channel, is fed to a sensor along with the noise *'k_o'* which is not correlated with the information signal. The signal *'$i + k_o$'* which is the combined signal forms the main input to the canceller. Another sensor takes noise *'k_1'*, uncorrelated with the message signal, as input. *'k_1'* is uncorrelated to the information signal but somehow correlated to *'k_o'*. The signal provided by this sensor serves as the reference to the canceller. The noise *'k_1'* is filtered to give an output *'x'* that tries to approximate the noise *'k_o'*. This output *'x'* is then subtracted from the input *'$i + k_o$'* to give the output of the entire system i.e. *'$y = i + k_o - x$'* [1].

Adaptive filters employ algorithms that allow the filter to 'adapt' according to the incoming signals.

The filter coefficients 'adapt' or get modified according to the signal.

There are the two different approaches or types of algorithms used in adaptive filtering:

a. Least Mean Square algorithms, based on Stochastic Gradient Approach
b. Recursive Least Squares algorithms, based on Least Square Estimation

Adaptive Filter Algorithms

1. LMS

LMS is a stochastic gradient descent method [8]. This method makes use of the gradient of the error curve. In this method, the filter only adapts or gets modified based on the error at the current time. The objective of this type of filters is to minimize the least mean squares of the error signal, which is the difference between the desired signal and the current actual signal [2]. The taps of the filter are adjusted by an extent proportional to the estimate of the error surface gradient at that instant.

It can be represented by the following equations.

$$\hat{Y}(n+1) = \hat{Y}(n) + \mu.u(n).e^*(n)$$

Where μ – *Step Size, u(n) – filter input, e*(n) – error signal*

2. NLMS

The conventional or 'pure' LMS suffers from a phenomenon called 'gradient noise' because of its random nature i.e. the samples at any instant can take any value. This can cause a big problem for large values of filter input. Large $|u(n)|$ can cause significant gradient noise amplification.

The solution to this problem is that input sample values can be normalised. As a result, the power of the input ends up being normalised. This modified technique is called Normalised Least Mean Squares, or NLMS.

$$\hat{Y}(n+1) = \hat{Y}(n) + \frac{\tilde{\mu}}{||u(n)||^2}.u(n).e^*(n)$$

3. RLS

At every instant, the Recursive Least Squares algorithm tries to accomplish an exact minimization of the sum of the squares of the desired signal estimation errors. [3]. The Recursive least squares (RLS) adaptive filter is an algorithm which, in a recursive manner, tries to compute the filter coefficients that would end up minimizing a weighted linear least squares cost function relating to the input signals. This is fundamentally different when compared to other algorithms such as the LMS that aims to minimize the MSE.[4]

For example, suppose that a signal *i(n)* has to be sent through a noisy channel, and as a result, it is received as:

$$x(n) = \sum_{k=0}^{q} b_n(k).i(n-k) + g(n+1)$$

Where g*(n)* is the additive noise. We recover the desired signal *i(n)* by using a j-tap FIR filter.

$$\hat{d}(n) = \sum_{k=0}^{j-1} x(n-k).f_n = \boldsymbol{f}_n^T.x(n)$$

Where

$$y_n = [y(n)\; y(n-1)\; y(n-2) \ldots\; y(n-p+1)]^T$$

Is the vector that contains the p latest sample values of *y(n)*. Our objective remains to try to compute or estimate the parameters of the filter ***f***, and at every time instant '*n*' we look at the newest least squares estimate by $\boldsymbol{f_n}$.

4. Kalman Filter

Kalman Filtering algorithm focuses on effectively minimizing the MSE(mean square error) recursively [3]. The advantage that Kalman Filter has over the above mentioned algorithms is that for computing the new state estimate in each cycle, only the previous state estimate is required along with the the new input data. This results in the system requiring comparatively lower memory [5]. It has been proven that the Kalman Filter has significantly better performance than the Wiener filtering methods for noise cancellation [6].

Final Simulations

We perform simulations using MATLAB. The results of the various algorithms are obtained and their performance is compared. Since their complexities are well known, a decision on which algorithm should be deployed in a specific scenario can be easily made depending whether performance or low computing complexity is the requirement from the system.

Simulation Results

1. LMS Results

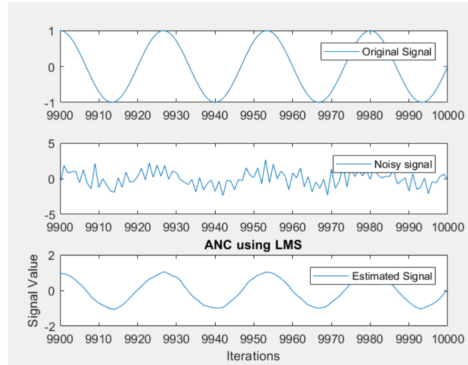


Figure 2. LMS Results

2. RLS Results

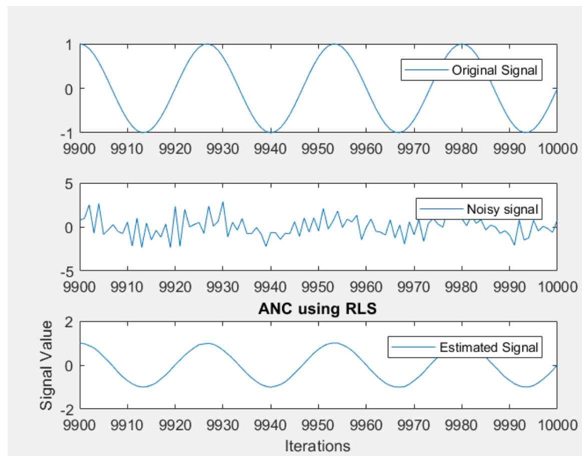


Figure 3. RLS Reults

3. Kalman Filter Results

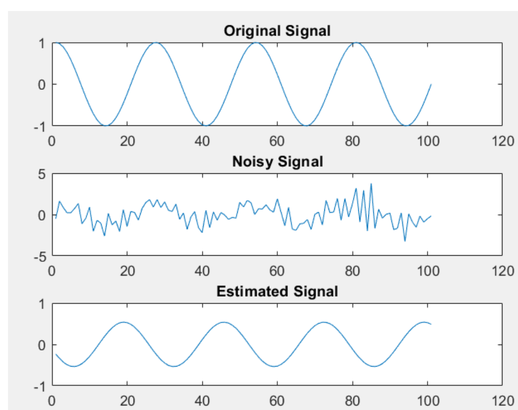
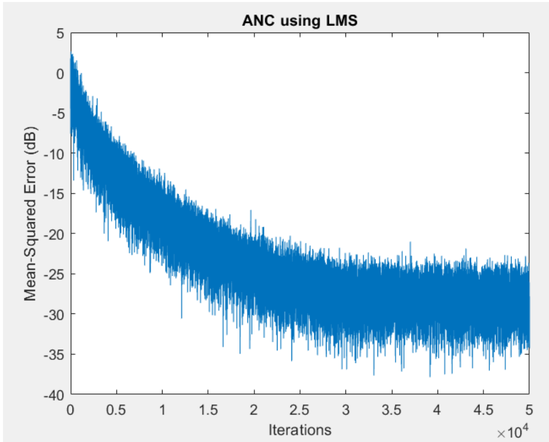
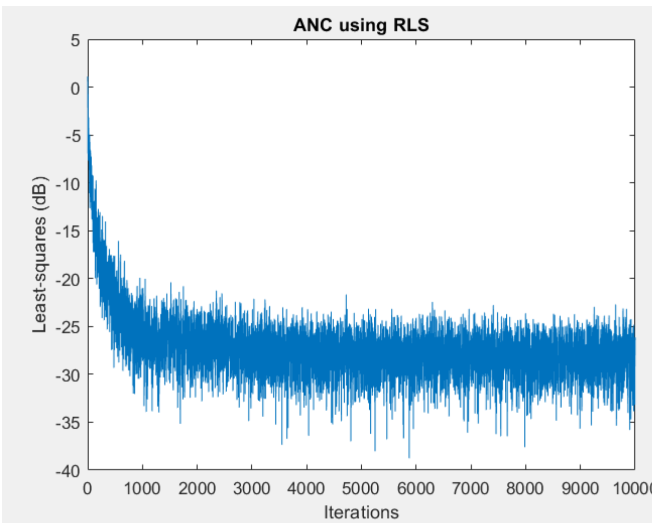


Figure 4. Kalman Filter Results

4. LMS VS RLS Convergence compared



(a)



(b)

Figure 5.

5. NLMS compared to LMS

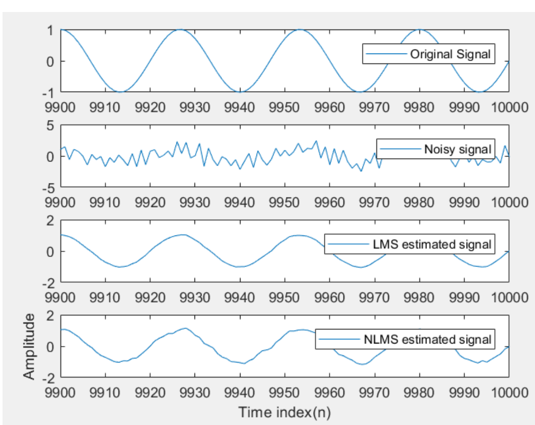


Figure 6

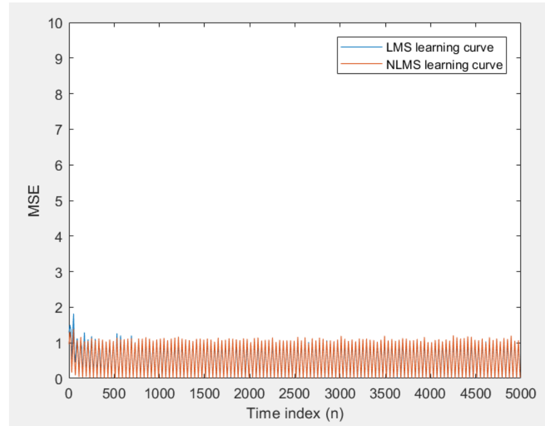


Figure 7

Analysis

The ability of all the employed adaptive algorithms to generate the original signal from the noisy signal can be observed from the simulations. Figures 2, 3 and 4 show the ability of LMS, RLS and Kalman Filter algorithms to reconstruct a basic sinusoid, but their comparative performance difference is not apparent. The fidelity with which each of them reproduces the original signal varies and can be inferred from the simulations. Figure 5 compares the convergence of LMS and RLS side by side. It can be then inferred that RLS is vastly superior to LMS, as in it converges significantly faster than LMS. The convergence rate is defined as the number of iterations required for the algorithm to converge to its steady state mean square error [9]. And from figure 5, we can clearly see that RLS has a much superior convergence rate as compared to LMS, as RLS converges to its steady state mean square error after around 1000 iterations whereas LMS takes about 3000 iterations to converge to its steady state. Another important observation is that while all algorithms reconstruct a signal that has lower power than the original signal, all algorithms except Kalman Filtering produce an output signal that has the same phase as the input signal. Only the Kalman Filter reconstructed signal has a phase shift of π

radians. From figure 6 it can be inferred that although, it is known that NLMS has a better convergence than LMS but for these particular signal values, the performance gap between the two is not that pronounced. Also, the main advantage that NLMS has over LMS is the fact that it isn't as affected by the over-powering gradient noise that comes into play when the sample values of the signal become very large. Since this is not the case here, therefore NLMS has no particular advantage over LMS in this scenario. In fact, the performance of NLMS is noticeably poorer in this scenario. Also, figure 7 highlights the comparison between LMS and NLMS MSE learning curves.

Conclusion

In conclusion, choice of the adaptive filter to be used heavily depends on the type of scenario, the performance requirement i.e. the convergence capability required and, and the computational capability available at hand. RLS is more superior to both NLMS and LMS, but has more computational complexity as compared to the two. Table 1 summarizes the computational complexities of the 3 algorithms.

Algorithm	**Complexity**	**Comparative Stability**
LMS	2K + 1	Less Stable
NLMS	3K + 1	Stable
RLS	$4K^2$	Highly Stable

Table 1. Computational Complexities and Relative Stabilities of various Adaptive Algorithms [7]

So, in scenarios permitting only limited computational complexity, LMS or NLMS should be chosen. In scenarios where performance is a priority, RLS should be employed.

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