

# Application of Markov Modeling in soap making plant: A Case study

Dr Mukesh Kumar

Deptt. of Mechanical Engineering from UIET,MDU Rohtak

---

**Abstract:** In the present work. The performance of the system is first estimated with the help of transition diagram or mathematical model based on Markov approach made pertaining to real working environment. The differential equations associated with the transition diagram are developed assuming that the failure and repair rate parameters of each component follow the exponential distribution. The long-run availability expression for the system has been derived with probabilistic approach using normalizing and boundary conditions. Further GA has been used to optimize the parameters affecting availability. MATLAB 7.4 is used for the analysis of the system.

---

## Introduction

The main purpose of availability evaluation is to forecast the probability that the system is operating at a specified time. A system is kept failure free under the given operative conditions to achieve the goal of production and long run availability. In the present period of mechanization and modernization, the task for setting up of production plants includes a terrific capital cost particularly for the industry like paper mills, food production industry; coal-fired thermal plants, butter oil processing plant, and textile factories, etc.

Kumar et al.[1] presented the concept of Maintenance Free Operating Period (MFOP) and developed performance models to predict MFOP for specifying reliability.

Abuelmaatti et al.[2] demonstrated simulated program with integrated circuit emphasis for the calculation of reliability, SSA and MTRF, of redundant systems. Fleming *et al.* [3] proposed a technology for forecasting the piping reliability with the help of new methods and database by using Markov piping reliability model. Cura [4] used the PSO approach in portfolio optimization. Garg et al.[5] proposed a mathematical model based on Markovian approach for a cattle feed plant were presented. Harish Garg et al.[6] had evaluated the industrial system fertilizer plant behavior using uncertain data Garg et al.[7] analyzed the system behavior by utilizing the rough and imperfect data of the complex repairable system.

Barabadi et al.[8] proposed an application of reliability models with covariates using spare parts requirements as reliability performance indicator and presented a case study of repairable system. Kumar et al.[9] discussed the SSA of packaging subsystem of paint industry. Kumar et al.[10]. implemented the PSO technique to improve the availability of a repairable system in lectogen milk powder system plant and to optimize the availability of various sub-systems. The analysis and performance modeling of Leaf Spring Manufacturing Industry has been discussed by Sharma et al.[11].Kumar et al.,[12] analyze the behaviour of multi-state repairable system of Towel Manufacturing System using G.A..Malik et al.[13] had examined performance modeling for water flow system. Ricardo Manuel Arias Velasquez et al.[14] had presented the analysis of maintainability, reliability, and availability for series capacitor bank.

In 2018, Wang et al.[15] had analyzed the reliability and availability for finding potential solutions of the hybrid cooling system. They proposed a Markov model to determine the performance of the system.

## System Description

Soap making Plant system of vital importance in the concerned industry. This system comprises of four subsystems namely Crutcher machine, Dryer machine, Plodder, and screw conveyor.

The mixture having a DFA (85%) and caustic (15%) fed to are being utilized for the raw material for soap manufacturing. this mixture is fed to the crutcher , where steam is also continuously supplied for sponification of fed material which results in sodium salt of fatty acids and water. After this neat soap mixture is supplied to spray dryer where moisture from mixture is removed. The function of plodder is to press the soap pieces together. These pieces are being are placed on screw conveyor to transfer them to another section of industry i.e. soap milling system.

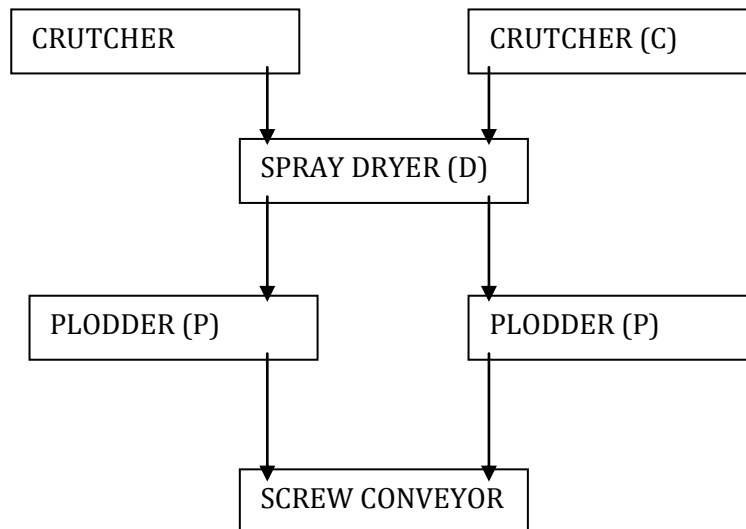


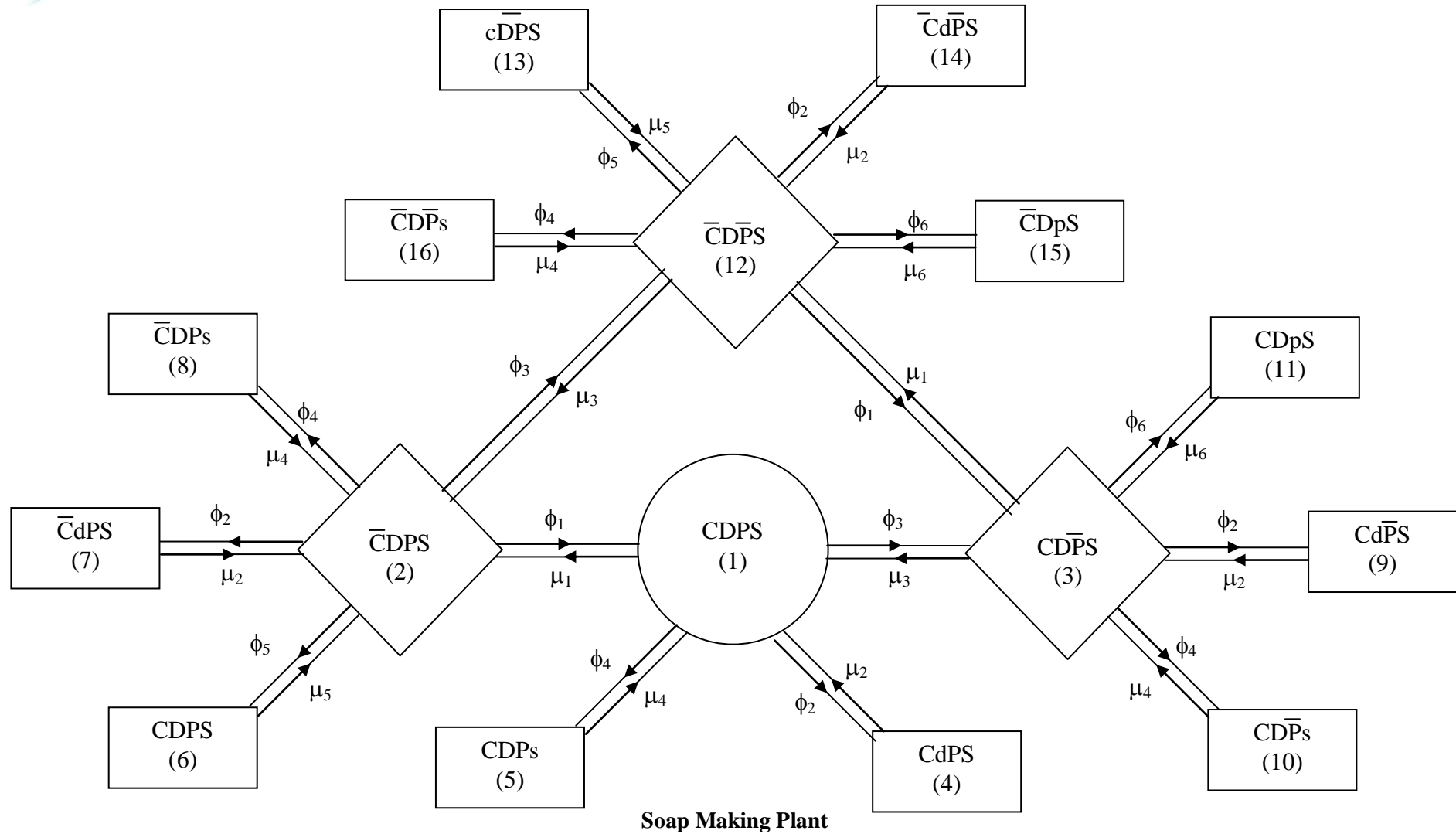
Figure1. Schematic flow diagram of Soap making Plant

### Assumption

- Failure/repair rates for every subsystem are exponentially distributed i.e. constant.
- No simultaneous failures occur between subsystems/system.
- The execution for a predetermined duration of a repaired unit is in the same class as new.
- No further failure can occur when system is in failure state
- The capacity and nature of standby subsystems are same as the working subsystems.
- All the subsystems are initially in good working state.
- At any given time each subsystem has three states viz. working, reduced or failed.
- System may operate in reduced capacity.

### Notations

C,D,P,S	Represents working state of Crutcher, Spray Dryer, Plodder, and Screw Conveyor.
C,d,p,s	Represents failed state of Crutcher, Spray Dryer, Plodder, and Screw Conveyor.
$\phi_1, \phi_2, \phi_3, \phi_4$	Represents failure rate of C,D,P,and S,
$\phi_1$	Represents failure rate of C <sup>1</sup> in reduced capacity state
$\mu_1, \mu_2, \mu_3, \mu_4$	Represents repair rate of C,D,P,and S
$\mu_1$	Represents repair rate of C <sup>1</sup> in reduced capacity state
P <sub>i</sub> (t)	Represents that probability of system in i <sup>th</sup> state at time 't'.
,	Represents Derivatives w.r.t. 't'
A <sub>V1</sub>	Steady State/Long Term Availability



**Performance modeling**

$$P'_1(t) + (K_1) P_1(t) = \mu_1 P_2(t) + \mu_3 P_3(t) + \mu_2 P_4(t) + \mu_4 P_5(t) \quad (1)$$

$$P'_2(t) + (K_2) P_2(t) = \mu_3 P_{12}(t) + \phi_1 P_1(t) + \mu_4 P_8(t) + \mu_2 P_7(t) + \mu_5 P_6(t) \quad (2)$$

$$P'_3(t) + (K_3) P_3(t) = \phi_3 P_1(t) + \mu_1 P_{12}(t) + \mu_6 P_{11}(t) + \mu_2 P_9(t) + \mu_4 P_{10}(t) \quad (3)$$

$$P'_{12}(t) + (K_4) P_{12}(t) = \mu_4 P_{16}(t) + \mu_5 P_{13}(t) + \mu_2 P_{14}(t) + \mu_6 P_{15}(t) + \phi_1 P_3(t) + \phi_3 P_2(t) \quad (4)$$

$$P'_3(t) + \mu_1 P_3(t) = \phi_1 P_1(t)$$

$$P'_4(t) + \mu_2 P_4(t) = \phi_2 P_1(t)$$

$$P'_5(t) + \mu_4 P_5(t) = \phi_4 P_1(t)$$

$$P'_6(t) + \mu_5 P_6(t) = \phi_5 P_2(t)$$

$$P'_7(t) + \mu_6 P_7(t) = \phi_2 P_2(t)$$

$$P'_8(t) + \mu_4 P_8(t) = \phi_4 P_2(t)$$

$$\mu_2 P_4 = \phi_2 P_1$$

$$\mu_4 P_5 = \phi_4 P_1$$

$$\mu_5 P_6 = \phi_5 P_2$$

$$\mu_6 P_7 = \phi_2 P_2$$

$$\mu_4 P_8 = \phi_4 P_2$$

$$\mu_2 P_9 = \phi_2 P_3$$

$$\mu_4 P_{10} = \phi_4 P_3$$

$$\mu_6 P_{11} = \phi_6 P_3$$

$$\mu_5 P_{13} = \phi_5 P_{12}$$

$$\mu_2 P_{14} = \phi_2 P_{12}$$

$$\mu_6 P_{15} = \phi_6 P_{12}$$

$$\mu_4 P_{16} = \phi_4 P_{12}$$

$$P'_9(t) + \mu_2 P_9(t) = \phi_2 P_3(t)$$

$$P'_{10}(t) + \mu_4 P_{10}(t) = \phi_4 P_3(t);$$

$$P'_{11}(t) + \mu_5 P_{13}(t) = \phi_5 P_{12}(t)$$

$$P'_{12}(t) + \mu_2 P_{14}(t) = \phi_2 P_{12}(t)$$

$$P'_{13}(t) + \mu_5 P_{13}(t) = \phi_5 P_{12}(t)$$

$$P'_{14}(t) + \mu_2 P_{14}(t) = \phi_2 P_{12}(t)$$

$$P'_{15}(t) + \mu_6 P_{15}(t) = \phi_6 P_{12}(t)$$

$$P'_{16}(t) + \mu_4 P_{16}(t) = \phi_4 P_{12}(t)$$

Where

$$K_1 = (\phi_1 + \phi_2 + \phi_3 + \phi_4)$$

$$K_2 = (\phi_3 + \mu_1 + \phi_5 + \phi_2 + \mu_4)$$

$$K_3 = (\phi_2 + \mu_3 + \phi_1 + \phi_4 + \phi_6)$$

$$K_4 = (\phi_6 + \phi_2 + \phi_4 + \phi_1 + \mu_1 + \mu_3)$$

With initial conditions at time  $t = 0$

$$P_i(t) = 1 \text{ for } i=1,$$

$$P_i(t) = 0 \text{ for } i \neq 1$$

**Steady State Behaviour:**

steady state conditions i.e. by putting  $t \rightarrow \infty$  and  $d/dt = 0$  on equations (1) to (4) we get:

$$\begin{aligned}
 (\phi_3 + \mu_1 + \phi_5 + \phi_2 + \phi_4) P_2 &= \mu_3 P_{12} + \phi_1 P_1 + \mu_4 P_8 + \mu_2 P_7 + \mu_5 P_6 \\
 (\mu_3 + \phi_2 + \phi_1 + \phi_4 + \phi_6) P_3 &= \phi_3 P_1 + \mu_1 P_{12} + \mu_6 P_{11} + \mu_2 P_9 + \mu_4 P_{10} \\
 (\mu_3 + \mu_1 + \phi_6 + \phi_2 + \phi_5 + \phi_4) P_{12} &= \mu_4 P_{16} + \mu_5 P_{13} + \mu_2 P_{14} + \mu_6 P_{15} + \phi_4 P_3 + \phi_3 P_2 \\
 \Rightarrow (\phi_3 + \mu_1 + \phi_5 + \phi_2 + \phi_4) P_2 &= \mu_4 P_{12} + \phi_1 P_1 + \mu_4 P_8 + \mu_2 P_7 + \phi_5 P_6 \\
 &(\phi_3 + \mu_1) P_2 = \phi_1 P_1 + \mu_3 P_{12} \\
 \Rightarrow (\mu_3 + \phi_2 + \phi_1 + \phi_4 + \phi_6) P_3 &= \phi_3 P_1 + \phi_1 P_{12} + \mu_6 P_{11} + \mu_2 P_9 + \phi_4 P_{10} \\
 &(\mu_3 + \phi_1) P_3 = \phi_3 P_1 + \mu_1 P_{12} \\
 \Rightarrow (\mu_3 + \mu_1 + \phi_6 + \phi_5 + \phi_5 + \phi_4) P_{12} &= \mu_4 P_{16} + \mu_5 P_{13} + \mu_2 P_{14} + \mu_6 P_{15} + \phi_4 P_3 + \phi_3 P_2 \\
 &(\mu_3 + \mu_1) P_{12} = \phi_4 P_3 + \phi_3 P_2
 \end{aligned}$$

$$\begin{aligned}
 P_{12} &= S_7 P_1 \\
 \text{Now } P_2 &= S_8 P_1 \\
 P_3 &= S_9 P_1
 \end{aligned}$$

$$\text{Now } P_4 = \frac{\phi_2}{\mu_2} P_1$$

$$P_5 = \frac{\phi_4}{\mu_4} P_1$$

$$P_6 = \frac{\phi_5}{\mu_5} P_2 = \frac{\phi_5}{\mu_5} S_8 P_1$$

$$P_7 = \frac{\phi_2}{\mu_2} P_2 = \frac{\phi_2}{\mu_2} S_8 P_1$$

$$P_8 = \frac{\phi_4}{\mu_4} P_2 = \frac{\phi_4}{\mu_4} S_8 P_1$$

$$P_9 = \frac{\phi_2}{\mu_2} P_3 = \frac{\phi_2}{\mu_2} S_9 P_1$$

$$P_{10} = \frac{\phi_4}{\mu_4} P_3 = \frac{\phi_4}{\mu_4} S_9 P_1$$

$$P_{11} = \frac{\phi_6}{\mu_6} P_3 = \frac{\phi_6}{\mu_6} S_9 P_1$$

$$P_{13} = \frac{\phi_5}{\mu_5} P_{12} = \frac{\phi_5}{\mu_5} S_7 P_1$$

$$P_{14} = \frac{\phi_2}{\mu_2} P_{12} = \frac{\phi_2}{\mu_2} S_7 P_1$$

$$P_{15} = \frac{\phi_6}{\mu_6} P_{12} = \frac{\phi_6}{\mu_6} S_7 P_1$$

$$P_{16} = \frac{\phi_4}{\mu_4} \quad P_{12} = \frac{\phi_4}{\mu_4} S_7 P_1$$

$$P_1 + P_2 + \dots + P_{16} = 1$$

$$P_1 =$$

$$\left[ \frac{1}{1 + S_7 + S_8 + S_9 + \frac{\phi_2}{\mu_2} + \frac{\phi_4}{\mu_4} + \frac{\phi_5}{\mu_5} S_8 + \frac{\phi_2}{\mu_2} S_8 + \frac{\phi_4}{\mu_4} S_8 + \frac{\phi_2}{\mu_2} S_9 + \frac{\phi_4}{\mu_4} S_9 + \frac{\phi_6}{\mu_6} S_9 + \frac{\phi_5}{\mu_5} S_7 + \frac{\phi_2}{\mu_2} S_7 + \frac{\phi_2}{\mu_2} S_7 + \frac{\phi_6}{\mu_6} S_7 + \frac{\phi_4}{\mu_4} S_7} \right]$$

$$= P_1 + P_2 + P_3 + P_{12} = (1 + S_7 + S_8 + S_9) P_1$$

### Performance Analysis:-

**Table 4.1 Decision Matrix of Crutcher machine (C) of Soap Making Plant.**

Table 4.1 represents the decision matrix for subsystem **Crutcher machine (C)**.as failure rate varies of subsystem (C) from 0.001 to 0.007(keeping other parameters constant) the availability drops by 0.72% & it gets improved by 0.16% with repair rate varies from 0.1 to 0.7.

$\mu_1 \backslash \phi_1$	0.001	0.003	0.005	0.007	Other Constant Parameters
0.01	0.9410	0.9378	0.9357	0.9342	$\phi_2 = 0.002, \mu_2 = 0.2,$ $\phi_3 = 0.003, \mu_3 = 0.4,$ $\phi_4 = 0.005, \mu_4 = 0.1,$
0.02	0.9420	0.9397	0.9380	0.9367	
0.03	0.9424	0.9406	0.9392	0.9381	
0.04	<b>0.9426</b>	0.9412	0.9400	0.9390	

**Table 4.2 Decision Matrix of Spray Dryer (D) of Soap Making Plant.**

Table 4.2 represents the decision matrix for subsystem **Spray Dryer (D)**.as failure rate varies of subsystem (D) from 0.002to 0.008 (keeping other parameters constant) the availability decreases by 2.74% & it gets improved by 0.56% with repair rate varies from 0.2 to 0.5.

$\mu_2 \backslash \Phi_2$	0.002	0.004	0.006	0.008	Other Constant Parameters
0.2	0.9410	0.9322	0.9236	0.9152	$\phi_1 = 0.001, \mu_1 = 0.01,$ $\phi_3 = 0.003, \mu_3 = 0.4,$ $\phi_4 = 0.005, \mu_4 = 0.1,$
0.3	0.9440	0.9381	0.9332	0.9265	
0.4	0.9455	0.9410	0.9366	0.9322	
0.5	<b>0.9463</b>	0.9428	0.9392	0.9357	

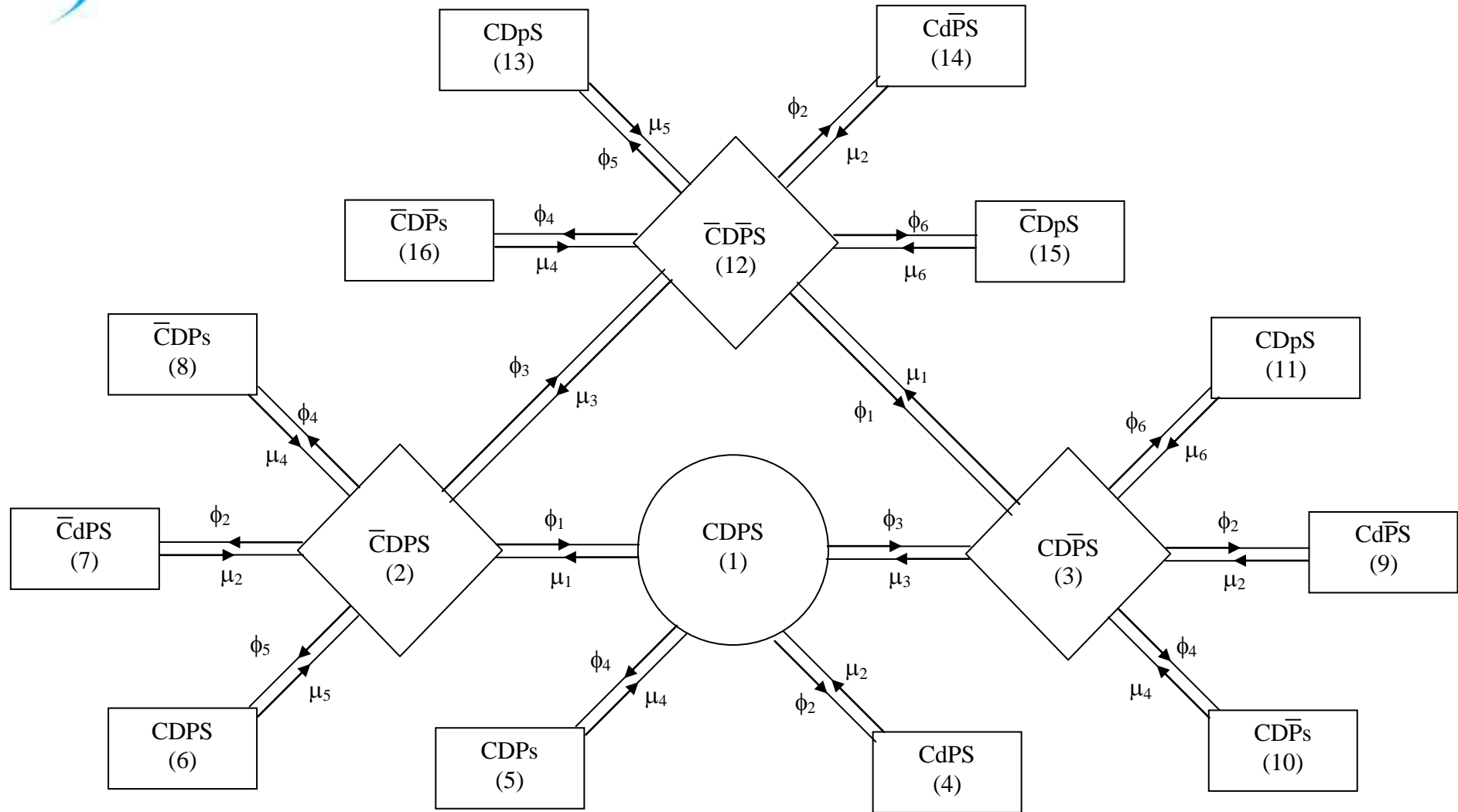
**Table 4.3 Decision Matrix of Plodder Machine(P) of Soap Making Plant.**

The component/machine number three does not affect the system availability significantly. Therefore it is not considered for the analysis.

**Table 4.4 Decision Matrix of Screw Conveyor Machine(S) of Soap Making Plant.**

Table 4.2 represents the decision matrix for subsystem **Screw Conveyor Machine(S)** as failure rate varies of subsystem (S) from 0.005 to 0.008 (keeping other parameters constant) the availability decreases by 2.74% & it gets improved by 3.52% with repair rate varies from 0.1 to 0.4.

$\mu_4 \backslash \Phi_4$	0.005	0.006	0.007	0.008	Other Constant Parameters
0.1	0.9410	0.9322	0.9236	0.9152	$\phi_1 = 0.001, \mu_1 = 0.01,$ $\phi_2 = 0.002, \mu_2 = 0.2,$ $\phi_3 = 0.003, \mu_3 = 0.4,$
0.2	0.9637	0.9591	0.9545	0.9500	
0.3	0.9715	0.9684	0.9652	0.9621	
0.4	<b>0.9754</b>	0.9731	0.9707	0.9684	



Transition diagram for Soap Making Plant



Performance optimization: (Table 1 and 2)

POP SIZE	10	20	30	40	50	60	70	80
$A_v$	0.8767	0.8827	<b>0.8838</b>	0.8836	0.8829	0.8828	<b>0.8824</b>	0.8821
$\phi_1$	0.0010	0.00101	<b>0.0010</b>	0.05006	0.00102	0.00165	0.001	0.00100
$\phi_2$	0.0069	0.00101	<b>0.00244</b>	0.03008	0.00204	0.00285	0.00206	0.002086
$\phi_3$	0.0030	0.002	<b>0.00304</b>	0.30480	0.00306	0.00822	0.00003	0.00003
$\phi_4$	0.005	0.00301	<b>0.00500</b>	0.49999	0.00503	0.00606	0.005004	0.00505
$\phi_5$	0.05	0.00505	<b>0.05007</b>	0.69999	0.05002	0.05429	0.00503	0.00500
$\phi_6$	0.0374	0.05007	<b>0.08999</b>	0.00101	0.08999	0.08001	0.003040	0.00305
$\mu_1$	0.1005	0.001	<b>0.31587</b>	0.00208	0.10529	0.29752	0.399999	0.39999
$\mu_2$	0.3568	0.002	<b>0.49999</b>	0.00300	0.49999	0.49666	0.799999	0.799999
$\mu_3$	0.652	0.003	<b>0.00100</b>	0.00507	0.69999	0.61698	0.89999	0.89999
$\mu_4$	0.04	0.005	<b>0.00205</b>	0.05004	0.03999	0.01452	0.39999	0.399999
$\mu_5$	.04	0.05	<b>0.0030</b>	0.08999	0.03999	0.03427	0.89999	0.899999
$\mu_6$	0.05	0.03	<b>0.00504</b>	0.23631	0.04999	0.04195	0.39999	0.399999

The simulation is done for utmost population size those changes from 10 to 80. Here the Generation size is kept constant as 500. The most favorable value of system's availability is 88.38%, for which the finest probable combination of failure and repair parameters is  $\phi_1=0.0010, \mu_1=0.31587, \phi_2=0.00244, \mu_2=0.49999, \phi_3=0.00304, \mu_3=0.00100, \phi_4=0.00500, \mu_4=0.00205, \phi_5=0.05007, \mu_5=0.0030, \phi_6=0.08999, \mu_6=0.00504$ , at population size 30 as given in table 1.

PAPULATION SIZE FIXED 100

POP SIZE	50	100	150	200	250	300	350	400
$A_v$	0.8813	0.8821	<b>0.8815</b>	0.8925	0.8878	0.8542	<b>0.8512</b>	0.8511
$\phi_1$	0.00156	0.00101	0.00107	0.00123	0.00202	0.00177	0.00508	0.00696
$\phi_2$	0.00366	0.00307	0.00204	0.00355	0.00457	0.00303	0.00221	0.00219
$\phi_3$	0.00741	0.003	0.00318	0.00301	0.00899	0.00894	0.00761	0.004967
$\phi_4$	0.00503	0.00502	0.00513	0.00503	0.00785	0.00665	0.00562	0.007955
$\phi_5$	0.05017	0.05019	0.05079	0.05002	0.05105	0.05319	0.05592	0.052392
$\phi_6$	0.07784	0.03065	0.089991	0.08801	0.08996	0.08586	0.08352	0.08395
$\mu_1$	0.28119	0.18138	0.344310	0.21025	0.26182	0.26156	0.35995	0.23424
$\mu_2$	0.49266	0.49963	0.49997	0.49999	0.21306	0.46869	0.21557	0.24062
$\mu_3$	0.68095	0.69999	0.69997	0.69961	0.69999	0.69999	0.69800	0.69732
$\mu_4$	0.02421	0.03999	0.03996	0.03999	0.02428	0.01190	0.01228	0.03026
$\mu_5$	0.039603	0.04	0.03999	0.03999	0.03999	0.03887	0.03998	0.03981
$\mu_6$	0.04756	0.05	0.04999	0.04999	0.04999	0.04723	0.04968	0.04993

Again, the simulation is made for maximum number of generation, varies from 50 to 400 with a step size of 50. Here, the population size is kept constant at 100. The optimum value of system's performance is 89.25%, for which the finest combination of failure and repair variable is  $\phi_1 = 0.00123$ ,  $\mu_1 = 0.21025$ ,  $\phi_2 = 0.00355$ ,  $\mu_2 = 0.49999$ ,  $\phi_3 = 0.00301$ ,  $\mu_3 = 0.69961$ ,  $\phi_4 = 0.00503$ ,  $\mu_4 = 0.03999$ ,  $\phi_5 = 0.05002$ ,  $\mu_5 = 0.03999$ ,  $\phi_6 = 0.08801$ ,  $\mu_6 = 0.04999$ , at generation rate 200 as given in table 2.

## References:

1. Kumar, u., Dinesh, Knezevic, j., and Crocker, j. (1999) "Maintenance free operating period an alternative measure to MTBF and failure rate for specifying reliability" *Reliability Engineering and System Safety* 64 (1999) 127-131
2. Abuelmaatti, M.T. and Qamber, I.S. 1. Abuelmaatti, M.T and Qamber, I.S (2000), "Using Spice Circuit Simulation Program in Reliability Analysis of Redundant Systems with Non-Repairable Units and Common-Cause Failures", *Active and Passive Elec. Comp.*, Vol. 22, No.4, pp. 235-255.
3. Fleming, K.N. (2004), "Markov Models for Evaluating Risk-Informed In-Service Inspection Strategies for Nuclear Power Plant Piping Systems", *Reliability Engineering and System Safety*, Vol. 83, Issue 1, pp. 27-45.
4. Cura, T. (2009) 'Particle swarm optimization approach to portfolio optimization', *Nonlinear Analysis: Real World Applications*, Vol. 10, No. 4, pp.2396-2406.
5. Garg, D., Singh, J. and Kumar, K. (2009), "Performance analysis of a cattle feed plant", *Journal of Science & Technology ICFAI*, Vol. 5 No. 2, pp. 83-94.
6. Garg H and Sharma S. P 2012 "Behavior analysis of synthesis unit in fertilizer plant", *International Journal of Quality & Reliability Management*, 29, (2), pp.217-232, 2012.
7. Garg, H. and Sharma, S. P. (2013), "Reliability-Redundancy Allocation Problem of Pharmaceutical Plant", *Journal of Engineering Science and Technology*, Vol. 8, pp. 190-18.
8. Barabadi A, Barabadi J and Markeset T (2014), "Application of reliability models with covariates in spare part prediction and optimization – a case study", *International Journal of Reliability Engineering System and Safety* Vol.123, pp. 1-7.
9. Mukesh kumar, Dr Vikas Modgil, Vineet Kumar, (2015) Mathematical Modelling and Availability Analysis of Packaging Section in a Paint Industry Case Study. *International Journal of Engineering Technology Science and Research* Volume 2 PP,179-185.
10. Mukesh kumar, Dr Vineet Kumar, Dr Vikas Modgil Availability Analysis and Optimization of Lactogen Milk Powder Production System using PSO, *International Journal of Mechanical Engineering and Technology*. Volume 8, Issue 11, November 2017, pp. 839-849.
11. Sharma, D., Kumar, A., Kumar, V. and Modgil, V. (2017), "Performance modeling and availability analysis of leaf spring manufacturing industry", *International Journal of Mechanical and Production Engineering*, Vol. 5, pp. 1-5.
12. Mukesh kumar, Dr vineet Kumar, Dr Vikas Modgil Optimization of Availability of Towel Manufacturing System: A case study. *International Journal of Applied Engineering Research (IJAER)* Volume 13, Number 12 (2018) pp. 10525-10534.
13. Malik S and Tewari P. C 2018 "Performance modeling and maintenance priorities decision for the water flow system of a coal-based thermal power plant", *International Journal of Quality & Reliability Management*, 35, (4), pp.996-1010.
14. Arias Velásquez R. M and Mejía Lara J. V 2018 "Reliability, availability and maintainability study for failure analysis in series capacitor bank", *Journal of Engineering Failure Analysis*, 86, pp.158-167, 2018.
15. Wang, J, Zhang, Q, Yoon S and Yu Y 2018 "Reliability and availability analysis of a hybrid cooling system with water-side economizer in data center", *Journal of Building and Environment*, pp.1-25.