

Noise Cancellation with RLS Adaptive Filter Algorithm

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Abstract - The proposed model works on the conceptual background of use of adaptive filters for noise cancellation in order to retain the signal quality. Adaptive filters are modifying in nature that is they alter their characteristics in order to accomplish their desired outcome. If the behavior (parameters) of the signal is known prior to the transmission in that scenario, fixed filters (low pass filter, high pass filter etc.) can be used but if the behavior of the transmitting signal is unknown in that case we need to use adaptive filters which can alter their characteristics (they can regulate their impulse response) and extract out the correlated signal. There are numerous kinds of algorithms used to carry out adaptive filtering like Least Mean Squares algorithm i.e. the LMS algorithm, averaging algorithm, Recursive Least squares i.e. the RLS and variations of RLS and LMS. In the proposed design noise entered in the system is removed by the use of RLS algorithm. This algorithm has its roots in the popular least squares method.

Keywords-LMS, RMS, adaptive filter

I. INTRODUCTION

Communication is a very important component of our lives. We communicate through various kinds of signals like speech signals, data transfer over the Internet, video conferencing and in lot of other ways. Noise is a form of electrical clutter which reduces the quality of our communication by interfering with the original signal. Noise can occur in a system both due to internal and external reasons and is broadly classified as internal noise and external noise. Internal noise mainly includes thermal noise (generated due to the random movement of free electrons because of heat) and shot noise (arises because of the distinct behavior of the electric charge) while the external noise consists of natural noise (due to lightning, solar noise) or manmade reasons (crosstalk). As noise degrades the standard of message signal it must be removed or at least should be brought down to a minimum level[1-8].

Noise cancellation can be done by generating an exact negative replica of the noise signal so that when these two signals are made to overlap the net effect of the noise signal is zero. It is easy to generate that negative copy of noise signal if we have knowledge about the input signal beforehand and also about the noise signal. But in most cases it is not available therefore we use adaptive filters which can generate a signal correlated to the input signal. Noise cancellation is done by the use of adaptive filters as they attempt to minimize the error[9-16]. Two mainly used algorithms for reducing the error are known as Least Mean Square algorithm also known as the LMS algorithm and Recursive least squares algorithm which is abbreviated as RLS algorithm. The objective of Least mean square algorithm is to obtain the filter parameters that can generate the least mean square of the error signal where the error signal represents the gap between the coveted signal and the original signal while RLS algorithm repetitively intends to find constants which minimize a cost

function calculated for weighted linear squares particularizing to the signals given as input.

In this paper we aim to propose a design which can minimize the noise and therefore can

help[17-22] in retaining the quality of the signal. We aim to do so by the use of RLS algorithm and by enhancing the performance of adaptive filter used.

II. PRINCIPLE OF THE PROPOSED MODEL

Noise is an unwanted signal that interferes with the original message signal in a communication setup. It occurs in a communication system due to various reasons:

Internal Noise

It occurs due to the random motion of electrons inside the electronic circuit. Electronic devices such as resistors, diodes and transistors are the major sources of internal noise.

Internal noise can be further classified into:

- Thermal (agitation) noise
- Shot noise
- Transit Time noise

External Noise

This is the noise generated due to man-made reasons or natural resources. They come from sources over which we have no control.

Industrial sources – generators, machines and fabricated equipment

Sources present in the atmosphere – lightning, static electricity.

The percentage of noise in a system is dependent on the bandwidth and temperature, the amount of current streaming within a component, the yield of circuit i.e. the gain, and the impedance offered by the circuit. Presence of noise in a system degrades the performance for both analog and digital systems. If the level of noise in the system is too high, then the receiver can't recover the original message signal from the noise corrupted message. Noise is calculated by determining the signal to noise ratio that is the SNR. Mathematically SNR is defined as

$$SNR = 10 \log(S/N) \text{ dB}$$

where S denotes the signal power whereas N denotes the noise power. Other important parameters related to noise are Noise Figure and Noise Factor.

For removal of noise we first need to quantize it or to represent it into a mathematical model. Noise factor F is defined by ratio of input SNR to ratio of output SNR i.e.

$$\text{Noise Factor, } F = (S/N)_{in} / (S/N)_{out}$$

In case of noiseless networks value of F is equal to 1 while generally $F > 1$.

Noise Figure is another numerical quantity related to noise. It basically represents the noise factor in decibels.

$$\text{Noise Figure } F \text{ dB} = 10 \log_{10} F$$

where F is greater than or equal to 0 dB.

The noise figure /factor is the yardstick which tells us how much a network degrades the $(S/N)_{in}$ & it has been found that a good network corresponds to a low value of F.

Various graphical models used for representing noise are Gaussian model, Rayleigh model, Exponential model, gamma/Erlang model and uniform model. In most of the communication systems noise is denoted as the Additive White Gaussian Noise (AWGN).

Components of AWGN Noise

Additive

Noise is normally additive in nature that is it sums up with the signal containing the message. Figure 1 shows model of the received signal with additive noise at the receiver output.

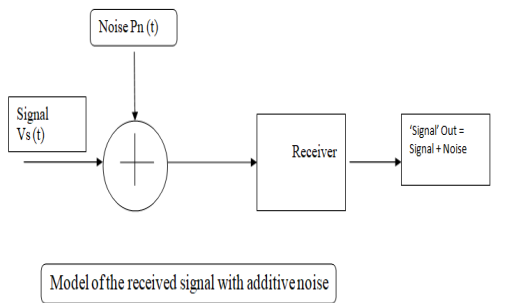


Figure 1 : Addition of noise in the input signal.

White

White noise denotes a numerical representation for signals and signal sources. It plays a significant role in structuring a model for wide sense stationary signals. A white noise process $\{W(t)\}$ is a random process that has constant power spectral density at all frequencies. Thus

$$S_w(w) = N_0/2, \quad -\infty < w < \infty$$

Where N_0 is known as the intensity of the white noise and is a real constant value.

III. ADAPTIVE FILTERS : A SPECIAL KIND OF FILTERS

Filter acts as a module that takes an input signal, does processing on it and after transformation returns an output signal that in some way changes the given input signal. For eg. If the input signal consists noise then we would want the filter to remove the noise and if there is no noise then the filter should leave the signal unchanged.

Filtering: refers to the process of deducing information about a value of interest by using data computed upto time 't' (including t also).

Classification and Characteristics of adaptive filter

Filters are divided into two categories: linear and non linear.

A filter is termed as linear if the value at the output of the device linear function of the information applied with filter input. Else the filter is nonlinear.

An adaptive filter is a type of digital filter with self altering properties that is it adjusts on its own, to variations in the input signals. They demand very less or almost no prior knowledge of the input signal and noise characteristics. If the system has characteristics like input signal narrowband and the noise broadband in nature, which in general is commonly what is seen, or the vice versa i.e. the noise is narrowband and the signal is broadband no prior knowledge is required by the adaptive filters, else they need a signal or aimed response i.e. correlated in some fashion to the signal to be approximated. They are non-linear and time variant in nature and are most useful when the signal conditions are changing non-rapidly. They rely on algorithms that run for fairly large number of iterations and these filters have adaptation algorithms for adjusting parameters in order to improve their performance. They are meant to administer the working environment and vary the filter transfer function accordingly. They can also trace the original input signal under non-stationary conditions. Adaptive filters are generally actualized either as a collection of scheduled instructions running on an arithmetical processing device such as a microprocessor or digital signal processing chip, or as a set of logic operations carried out on a specially made VLSI circuit or on a programmable gate array chip. Nonetheless, neglecting any flaws inducted by computational precision effects in these applications, the basic functioning of an adaptive filter can be described independently of the particular physical implementation that it takes. Due to this, we need to pay heed to the numerical forms of adaptive

filters and not to their implementations/executions in S/W or H/W. An adaptive filter is characterized by the following properties:

1. Processing of signals by the adaptive filter.
2. The model that explains how we mathematically arrive at the output from the input.
3. The specifications of this model that can be repetitively modified to change the filter's input-output dependency.
4. The adaptive algorithm used that explains how the guidelines are accustomed from one time instant to the next.

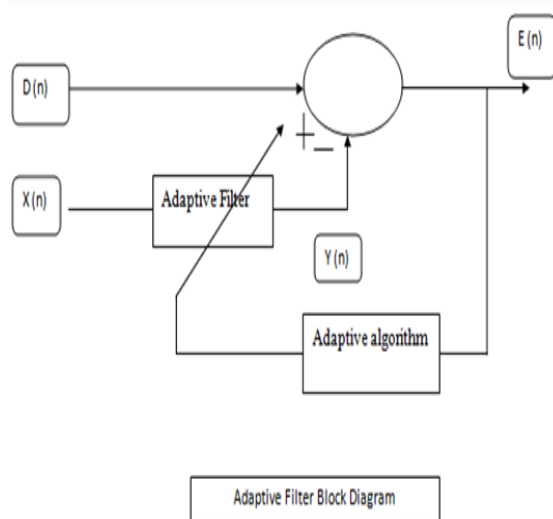


Figure 2 : Basic block Diagram of Adaptive Filter
 The fundamental processes of an adaptive filter are :

1. Filtering process: The outcome of this process is an output signal in correspondence to an input signal.
 2. Adaptation process: This process aims for adjusting the specifications i.e. the filter parameters to the environment.
- Both these processes are shown in the Figure 2.

Applications of Adaptive Filters:

1. Noise Cancellation: They remove noise from the received signal adaptively to enhance Signal to Noise (SNR) ratio.
2. Signal Prediction: They are used to provide a prior estimation of the present value of a random signal.

3. System Identification: They can provide approximations for systems whose behavior is not known to us.
4. Echo Cancellation: They are also used to eradicate unidentified intrusion by a fundamental signal.

ALGORITHMS FOR ADAPTIVE FILTERING

A substantial no. of adaptive algorithms have already been created and improvised to carry out the working of adaptive filters. Any algorithm that is being developed should possess the following characteristics:

1. It should be realizable i.e. practical to implement.
2. It should adjust itself to the environment so that it can find the best suited coefficient in order to minimize error.
3. It should provide the performance which we want.

Various algorithms used are:

1. Least means squares (LMS) algorithm
2. Normalized Least means squares (NLMS) algorithm
3. Recursive Least Squares (RLS) algorithm
4. Affline Projection Algorithm

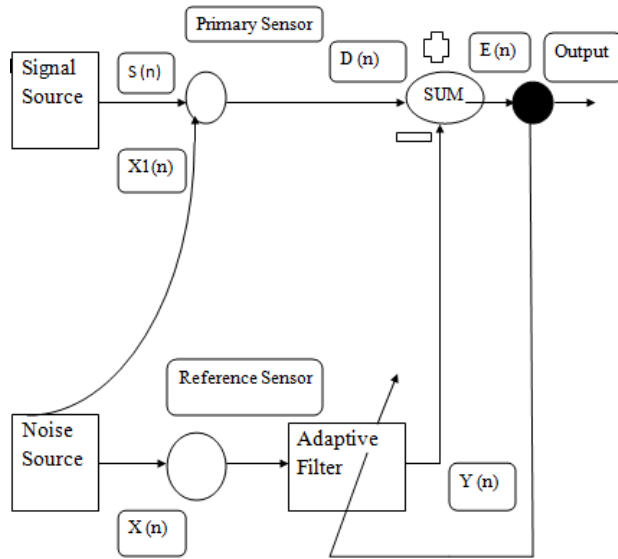
Above all of the algorithms LMS and RLS are the ones which are most commonly used.

LMS (Least Mean Squares) Algorithm:

- A.It alters the weight $w(n)$ of the filter.
- B.It adaptively accustoms to the filter taps according to the equation:

$$w(n+1) = w(n) + \mu * e(n) * x(n)$$

- C. It uses negative feedback to minimize the value of error signal.
 - D. It is sturdy (robust) in nature.
 - E. It is slow in convergence and is affected by the changes in step size parameter.
 - F. It requires number of iterations equal to the dimension of the input signal.
- LMS Algorithm Steps :



LMS ADAPTIVE FILTER ALGORITHM BLOCK DIAGRAM

Figure 3 : Block Diagram for LMS Adaptive Filter Algorithm

Each cycle of LMS includes three steps which is shown in Figure 4 below:

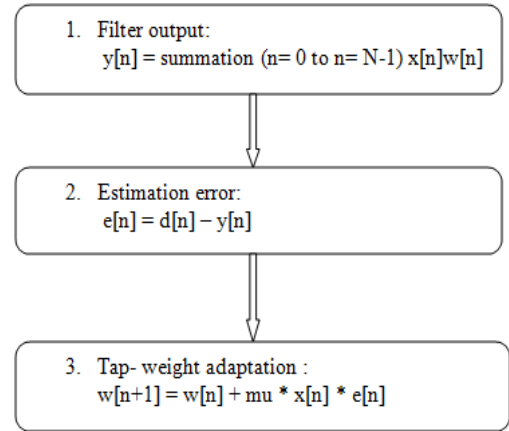


Figure 4:Steps involved in each iteration of LMS Adaptive Filter Algorithm

Stability Condition:

For filter to have stability: $0 < \mu < 2/\text{input signal power}$

Larger values for step size :

It increases the adjustability rate (i.e. faster adjustability)

It also increases the residual MSE(mean squared error).

Advantages of LMS over other algorithms:

It is simple to realize .

It provides stable and robust performance against various values and conditions of signals.

Disadvantages of LMS

It has a slow convergence rate.(i.e the time and the number of iterations taken to reach to thre optimal solution is more).

RLS Algorithm

It recurrently searches for the filter constants or coefficients in order to minimize a cost function calculated for weighted linear squares particularizing to the signals given as input.

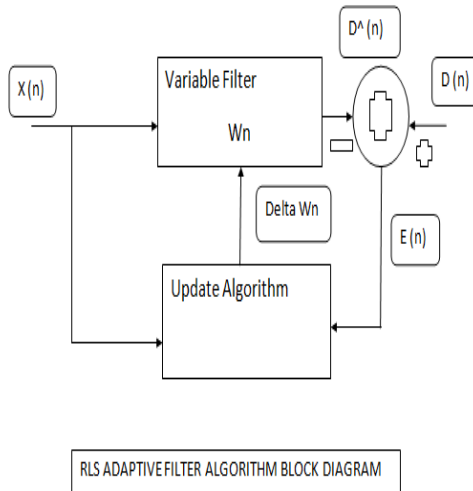


Figure 5 : Block Diagram for RLS Adaptive Filter Algorithm

Figure 5 shows the block diagram used for realizing the RLS adaptive filter algo. In this the variation of filter tap weight vector is governed by the equation:
 $w(n) = w(\text{transpose})(n-1) + k(n) * e(\text{bar})(\text{subscript } n-1)(n)$.

The conceptual background of the RLS algorithm which is the least squares method is a numerical layout for evaluating the most appropriate trajectory for a given collection of values. This curvature is achieved by reducing the overall total of squares of deviations in values from the trajectory, to a minimum value. The RLS algorithm recursively solves the least squares problem. The RLS algorithm equation involves the use of constants λ and δ that denote the forgetting factor and regularization parameter respectively. The value of forgetting factor is always greater than zero and less than unity, which basically tells us the remembering capacity of the algorithm; and the regularization parameter's value is measured with the help of SNR of the signals. The vector $w(\text{cap})$ denotes the filter's weight vector and the matrix P denotes the inverse correlation matrix. The vector p_i is used as an emissary step to calculate the gain vector k and when the value of k equal to one marks the time instant of initiation of RLS algorithm. In order to restore the value of weights, the gain factor is multiplied by $\zeta(n)$ where the $\zeta(n)$ is factor which takes into account the predetermined estimation error. After the multiplication the resultant of

multiplication is summed up with the weight vector in order to restore the value of weights to its correct value. Once the weights have been revised the value of inverse correlation matrix is again determined & the new cycle starts using the recent values of input. As the value of forgetting factor is less than one more significance is provided to the most current error estimates and therefore the more current input samples which leads to an arrangement which puts more importance on newly received values of information which we are attending to and not remembers the previous values.

We need to diminish the value of cost function of RLS.

$$C(n) = \sum_{i=1}^n \beta(n,i) e(i)^2$$

Where $C(n)$ is the cost function of the RLS algorithm & $\beta(n, i)$ is the weighing vector and its value lies between 0 and 1 and the value of 'i' varies from 1 to n and $e(n)$ is the Measure of the error signal & $e(i) = d(i) - \omega^H(n)u(i)$.
 Also $\beta(n, i) = (\lambda)^{(n-i)}$ where $\lambda \rightarrow$ Forgetting Factor.

Figure 6 : Equation of the cost function for RLS algorithm.

Figure 6 shows the cost function equation of the RLS algorithm. It represents the equation in terms of forgetting factor and error signal value.

To update the cost function regularization is done:

A regularizing term:

$$\delta * (\lambda)^n * \|\omega(n)\|^2 = \delta * (\lambda)^n * \omega^H(n) * \omega(n)$$

After regularization:

$$C(n) = \sum_{i=1}^n (\lambda)^{(n-i)} e(i)^2 + \delta * (\lambda)^n * \|\omega(n)\|^2$$

Sum of weighted error squares:

$$\sum_{i=1}^n (\lambda)^{(n-i)} e(i)^2 = \sum_{i=1}^n (\lambda)^{(n-i)} |d(i) - \omega^H(n)u(i)|^2$$

If $\varphi(n)$ is assumed to be correlation matrix of input $u(i)$ then

$$\varphi(n) = \sum_{i=1}^n (\lambda)^{(n-i)} * u(i) * u^H(i) + \delta * (\lambda)^n * I$$

And average cross correlation vector $z(n)$ is

$$z(n) = \varphi(n) * \omega^A(n) \text{ Where } n \text{ ranges from } 1 \text{ to infinity}$$

Figure 7 : Equation of the cost function after regularization and the weighted error squares sum
 Figure 7 shows the cost function equation of the RLS algorithm after regularization. It also depicts the equation of sum of weighted error squares and that of average cross correlation vector.

If the matrix inversion lemma is considered then inverse of correlation matrix can also be found:

$$\varphi^{-1}(n) = P(n)$$

Cost function is represented in terms of gain vector where $K(n)$ is the gain vector

$$K(n) = P(n) * u(n) = \varphi^{-1}(n) * u(n)$$

Tap weight vector: $\omega^A(n) = \varphi^{-1}(n) * z(n)$

Summary of RLS:

$$K(n) = \frac{\pi(n)}{\lambda + u^H(n) * \pi(n)}$$

$$\text{Where } \pi(n) = P(n-1) * u(n)$$

$$Zeta(n) = d(n) - \omega^A(n-1) * u(n)$$

$$\omega^A(n) = \omega^A(n-1) + k(n) * Zeta(n)$$

$$P(n) = \lambda^{-1} * P(n-1) - \lambda^{-1} * k(n) * u^H(n) * P(n-1)$$

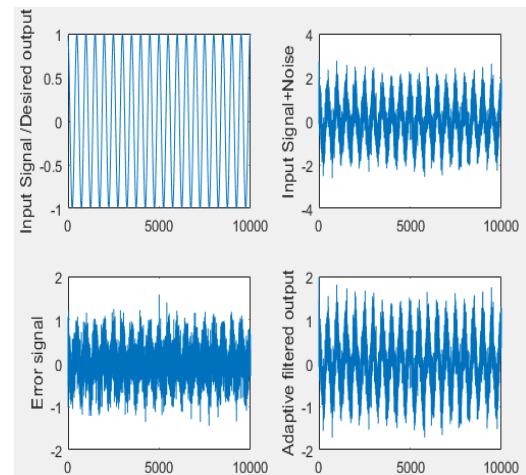
Figure 8 : Equation of the cost function in terms of gain vector and the summary equations of RLS algorithm. Figure 8 shows the cost function equation of the RLS algorithm in terms of gain vector. This also contains the equations summarizing the RLS algorithm. Advantages of RLS Algorithm :

Convergence rate of RLS is much higher in comparison to LMS, NLMS and APA. The noise cancellation capability of RLS algorithm is the highest among all the algorithms.

Disadvantages of the RLS Algorithm:

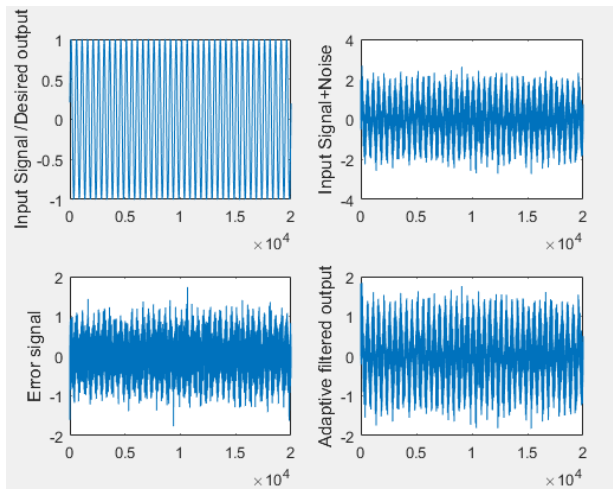
It is the most complex algorithm of all the four algorithms.

VI. RESULTS

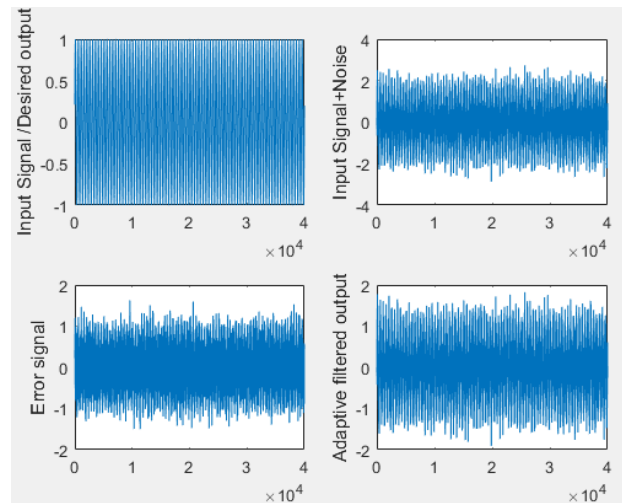


When length of input sequence = 10000

Figure 9 (a) : Graph of the input signal, noise mixed input signal, error signal and the filtered output signal for length of input signal equal to 10000.

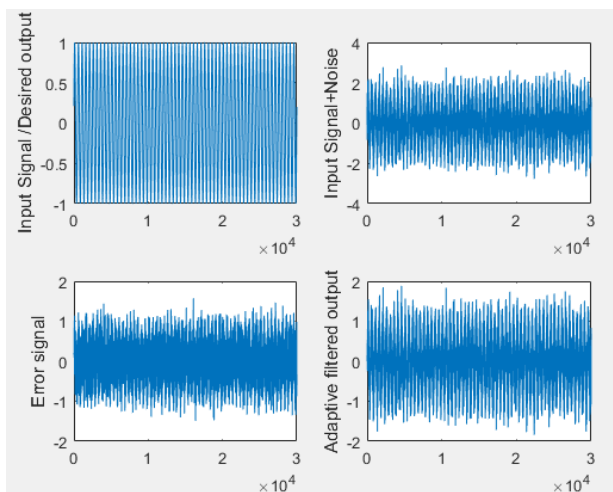


When length of input sequence = 20000



When length of input sequence = 40000

Figure 9 (b)



When length of input sequence = 30000

Figure 9 (c)

The figures (Figure 9(a),(b),(c),(d)) shown in the results section show the graphs of the input signal, noise mixed input signal, error signal and the filtered output signal for various lengths of input signal.

Figure 9 (d)

V.CONCLUSION AND FUTURE SCOPE

In this report we have seen that adaptive filter algorithms are used to perform noise cancellation. While the LMS algorithm is easier to implement and robust in nature it takes more time whereas the RLS algorithm has lesser convergence time than LMS algorithm but it requires more computational complexity. This noise cancellation property of adaptive filters can be used to build better telephonic models of communication and remove the noise in speech, audio and video signals. It can also be used to build better hearing aids and remove any noise present. It can be used to process satellite images and images which are corrupted by noise. Apart from that it can be used for purposes like developing noise cancellation headphones. As we can deduce from the results that some amount of error is still present in the processed signals. So better algorithms can be developed to reduce the percentage of noise in the processed signal. Also various variants of the already developed algorithms can be combined together to develop an algorithm which provides us better processing result and less computational complexity. As the power of computation increases day by day, more complex algorithms can also be developed in trade of almost completely non-interfered signal.

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