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# Modelling of Dynamic RCS of Radar Targets for Discrimination

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**Abstract:** The Radar Cross Section (RCS) of complex shaped targets varies significantly with viewing angle and the motion of the targetmaking RCS a fluctuating quantity, referred to as the dynamic RCS. Target identification techniques, reported in literature so far, use the static RCS. In a real scenario, it is the dynamic RCS that needs to be considered. For computing the fluctuation characteristics, the dynamic RCS of the target is modelled using a distribution of monostatic RCS obtained at various aspect angles around the mean. The Gaussian, lognormal and Weibull distributions which are bi-parametric are used for modelling the dynamic RCS. In this paper, the effect of using the dynamic RCS, in discriminating two model aircraft with a minor structural variation is studied. It is shown that the discrimination algorithm proposed by the authors, is robust for aspect angle changes when the dynamic RCS is considered. The algorithm works well for a spread of aspect angle variations of more than 30° for Gaussian distributions. The range of aspect angle variations for proper discrimination has reduced to 20° and 10°, when the Weibull and lognormal distributions respectively are used which are well within the deviations expected in practical situations.
**Keywords:** Discrimination of targets, dynamic radar cross section, fluctuating RCS, probability distribution.

## I. Introduction

The Radar Cross Section (RCS) of a target is a measure of the reflected electromagnetic wave back to the radar. It depends on parameters such as the transmitted wavelength, target geometry, orientation and reflectivity [1]. In practical radar systems, the radar and the target are in relative motion causing a fluctuation in the RCS. The fluctuations in the RCS could arise due to temporal (vibrations, wind) and spatial (aspect changes due to rotation and translation-velocity) variations [2]. The RCS that fluctuates in amplitude and/or in phase is referred to as the dynamic RCS. The amount by which the fluctuating and the non-fluctuating RCS differ depends on the rapidity of the fluctuations and on the statistical distribution of the fluctuations [7]. It is reported that a large real sized aircraft may exhibit 10 to 15 dB of RCS fluctuations, for aspect changes of a fraction of a degree [1].

Targetdiscrimination methods reported in literature [4-6], are implemented using the static RCS. However, as the real-time measured RCS of the target is statistical in nature, it is more appropriate to consider the dynamic RCS for discrimination. The actual dynamic RCS of the target should be obtained by measuring the RCS of the target in flight. However, for analysis purposes, this is cumbersome. To overcome this problem, the dynamic RCS of a target is modelled using statistical models such as the Swerling, lognormal, Rayleigh, Weibull etc. [12].

The RCS models adopted should be simple and should adequately represent the real data. Fluctuation models such as the Rayleigh model are based on a single parameter and are found to be inadequate in fitting the data. The Swerling models I-IV, are applicable for both time variations of RCS, that is pulse to pulse fluctuations (rapid fluctuations), and spatial variations, i.e. scan to scan fluctuations (slow fluctuations). They are special cases of the chi-square and Rayleigh models. Models such as chi-square, Weibull, lognormal and k-distributions are bi-parametric, and they match data for mean, and standard deviation which controls the shape of the density distribution curve [2]. The dynamic RCS of a ballistic missile has been simulated by modelling the fluctuation as a multiplicative noise with a lognormal distribution and is reported in literature. The difference between the static and dynamic RCS characteristics for detection has been analyzed, emphasizing the necessity of considering the dynamic RCS [13].

A resonance based frequency domain technique to discriminate between targets with minor variation was developed by the authors [8]. The algorithm quantifies the discrimination by a number called 'Risk' which uses the direct amplitude returns from the target. The main focus of this paper is to analyse the effect of dynamic RCS on discriminating two similar targets with minor structural variation. For illustrative purposes, the discrimination of a Perfectly Electrically Conducting (PEC) aircraft of 1m fuselage length, with and without a minor variation is considered. The dynamic RCS of the target is modelled by determining the expected value of the RCS obtained for a range of aspect angle variations, using the Gaussian, lognormal and Weibull distributions. Although, the factors contributing to RCS fluctuations are many, for simplicity, only the effect of fluctuations caused by aspect angle variations in the azimuth plane is considered. The maximum aspect variation (deviation) within which the discrimination technique works successfully as in the non-fluctuating case is determined and presented for each model.

The paper is organised as follows. In section II, the Gaussian, lognormal and Weibull distributions are presented in brief. The method of computing the dynamic RCS using different distributions are discussed and the discrimination function Risk is also presented. In section III, the modelling of dynamic RCS for aircraft model considered and the results obtained in discriminating aircraft with minor variation are presented. The conclusions drawn from this work are presented in section IV.

## II. Formulations

A. Gaussian, Lognormal and Weibull distributions [2]

The Gaussian distribution function is the most common distribution, and is applicable to most natural phenomena. The distribution is described as

$$w(\phi) = \frac{1}{\sigma\sqrt{2\pi}} \quad e^{-\frac{(\phi-m)^2}{2\sigma^2}} \tag{1}$$

where, $w(\phi)$ is the Probability Density Function (PDF) for variation in $\phi$ and is a function of parameters $\sigma$, the standard deviation and m, the mean.

The PDF of lognormal distribution is given by (2) as

$$w(\phi) = \frac{1}{\phi\sigma\sqrt{2\pi}} e^{-\frac{-(\ln(\phi)-m)^2}{2\sigma^2}} \tag{2}$$

The normal and lognormal distributions are closely related. The random variable $\phi$ is distributed log normally with parameters m and $\sigma$, when $\ln(\phi)$ is distributed normally with mean m and standard deviation $\sigma$.

Weibull distribution is a two parameter model. The PDF is given by (3)

$$w(\phi) = \frac{b}{a}\left(\frac{\phi}{a}\right)^{b-1} e^{-\left(\frac{\phi}{a}\right)^{b}} \tag{3}$$

where, 'b' is the shape parameter and 'a' is the scale parameter.

B. Modelling of dynamic RCS

The monostatic frequency response of a target is a function of frequency, elevation and azimuth angles and is represented as $RCS_{static}$ (f, θ, ϕ). In this study, only the spatial variation along the azimuth $\phi$ is considered. Thus, dynamic RCS is now a function of f and $\phi$, for a given θ. A set of static $RCS_{static}$ is obtained at discrete angles between $\phi_{N-k}$ and $\phi_{N+k}$, where $\phi_N$ represents the mean angle and 'k' represents the deviation in angle on either side of $\phi_N$. Weights $w(\phi)$ are generated using one of the distributions described in section IIA. The expected values of RCS at each frequency f are computed using (4), to obtain the $RCS_{dynamic}$ $(f, \phi_N)$.

$$RCS_{dynamic}\ (f, \phi_N) = \frac{\sum_{i=N-k}^{N+k} w(\phi_i) RCS_{static}\ (f, \phi_i)}{\sum_{i=N-k}^{N+k} w(\phi_i)} \tag{4}$$

The $RCS_{dynamic}$ obtained in (4) is used in the discrimination algorithm and is discussed below in section IIC.

C. Discrimination of targets using the dynamic RCS

A resonance based frequency domain algorithm is used to discriminate two closely resembling objects [8]. The algorithm involves determining the Risk factor $R_{td}$ [9] between two targets - unknown (target) and known (database) and is defined as

$$R_{td} = \int_{\omega_a}^{\omega_b} \left\{\frac{d^3}{d\omega^3}[|D_d(j\omega)|^2 . A_t(\omega)]\right\}^2 d\omega \tag{5}$$

The distinction polynomial $D(j\omega) = \prod_n (s - a_n)$ in (5), represents the database target which is constructed using the true Natural Resonant Frequencies (NRFs) $a_n$ of the target. $A_t(\omega)$ is the frequency response of the unknown target, within the frequency range $\omega_a$ to $\omega_b$ at a particular aspect angle. Similarly, the risk factor $R_{dd}$ is defined wherein $A_t(\omega)$ is replaced by $A_d(\omega)$, the frequency response of the database target. Finally, the discrimination is quantified by normalizing Rtd with the database target risk factor $R_{dd}$ (6) and computing 'Risk' in decibels as in (7).

$$\frac{R_{td}}{R_{dd}} = \frac{\int_{\omega_a}^{\omega_b} \left\{\frac{d^3}{d\omega^3}[|D_d(j\omega)|^2 . A_t(\omega)]\right\}^2 d\omega}{\int_{\omega_a}^{\omega_b} \left\{\frac{d^3}{d\omega^3}[|D_d(j\omega)|^2 . A_d(\omega)]\right\}^2 d\omega} \tag{6}$$

$$Risk_{in\ dB} = 10\log_{10}\frac{R_{td}}{R_{dd}} \tag{7}$$

In defining Risk, $A_t(\omega)$ and $A_d(\omega)$ are the static RCS of the unknown target and database target respectively. The distinction polynomial $D_d(j\omega)$ and $A_d(\omega)$ are parameters stored in the database. $A_t(\omega)$ is the direct amplitude return, received from a target which needs to be discriminated. It is proposed to substitute the static RCS of the unknown target $A_t(\omega)$ with its dynamic RCS, in this study. The effect of dynamic RCS on the discrimination is analyzed.

## III. Discrimination of PEC aircraft models

A. Modelling the dynamic RCS of a PEC aircraft

A PEC aircraft model T of 1m length is shown in Fig.1. A minor variation (mv) in the structure is introduced by increasing the distance a1 from 0.4m to 0.5m and this target is referred to as target Tmv. The dimensions of the aircraft model are given in table-1. The targets are modelled using CADFEKO and the monostatic RCS of the two targets is computed using the Method of Moments solver in FEKO [11].

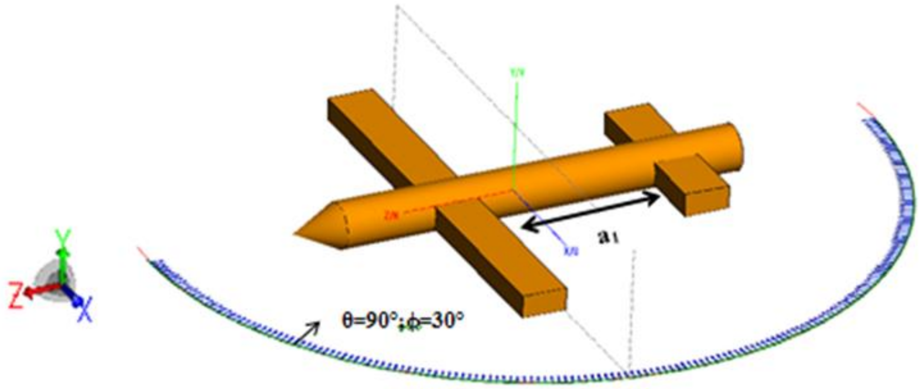


Figure 1. PEC aircraft model and schematic view of directions at which the static RCS are computed

Table 1.Dimensions of the aircraft model [10]

| Parameters | Value in meters |
|---|---|
| Length of aircraft | 1 |
| Length of front wing | 0.92 |
| Length of tail wing | 0.38 |
| Distance between front and tail wing a1 | |
| T | 0.4 |
| Tmv | 0.5 |
| Placement of front wing from head | 0.2 |
| Width of the wings | 0.1 |
| Thickness of wings | 0.05 |
| Radius of fuselage | 0.05 |
| Nose height | 0.1 |

The procedure of determining the NRFs of a database target, and building of the distinction polynomial is detailed in [10]. To study the effect of dynamic RCS on discrimination, the static RCS of T and $T_{mv}$ are first computed at discrete angles keeping $\theta = 90°$ constant, and $\phi$ is varied from 0° to 180° in steps of 1°.

A study of the variation in RCS in the azimuth plane, revealed that the maximum change occurred around an incident angle of $\phi$=30°. The static RCS varied by as much as 50% in peak amplitudes with aspect angle changes of ±5° around $\phi$ =30°. The amplitude changes observed around $\phi$=0°,90° and 180° aspects were much smaller and ranged between 1-2%. Hence, $\phi$ =30$^0$ has been chosen for illustration in this paper. The RCS obtained at discrete angles from 25° to 35° around $\phi$ =30° for target Tmv is shown in Fig.2. A similar variation in frequency response is expected at other aspect angles.

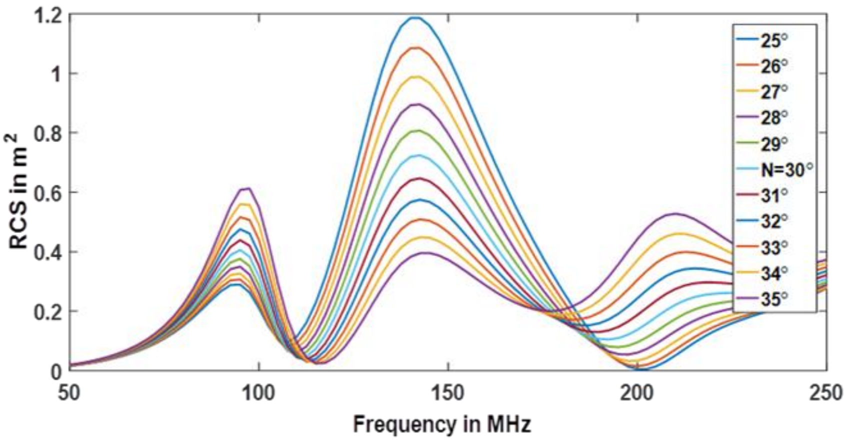


Figure 2. Static RCS of aircraft Tmv

B. Discrimination results and discussions

In the discussions below, the database target and the structurally modified target are notated as T and Tmv when modelled with static RCS, and notated as T* and Tmv* when modelled with the dynamic RCS.

To study the effect of dynamic RCS on discrimination, the following cases are considered.

1)Discrimination of target T with T : R(T,T)
2)Discrimination of target T* with T : R(T*,T)
3)Discrimination of target Tmv with T : R(Tmv, T) and
4)Discrimination of target Tmv* with T : R (Tmv*,T )

The static RCS of targets T and Tmv at aspects ranging from $\phi$=0:60° in 1° steps are computed. The dynamic RCS is modelled using the Gaussian, lognormal, and Weibull distributions separately for mean angle 30$^0$ and for standard deviations of $\sigma$ = 1°, 2°, 3°, 4°, 5°, 15°, 20°, 25°, and 30°.

The discrimination results obtained for cases 1 and 2 using the three distribution models are shown in Fig.3.

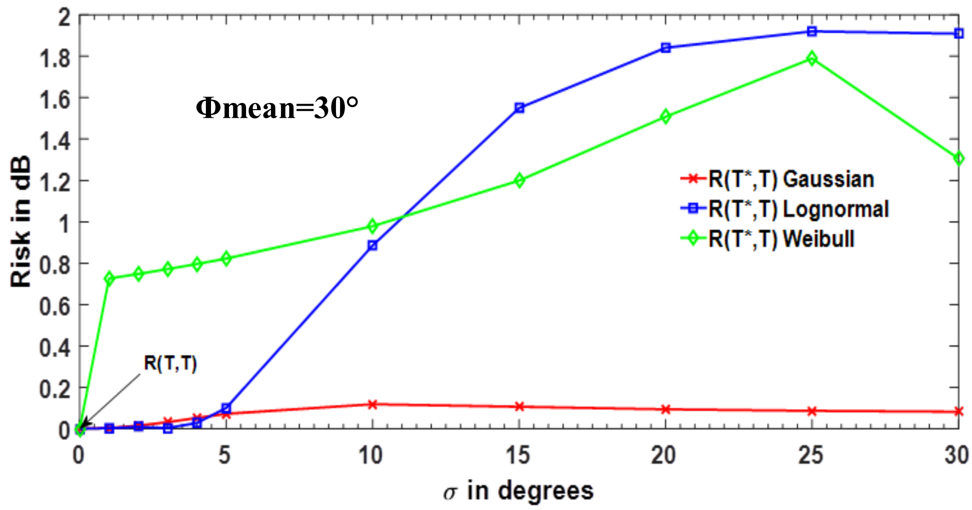


Figure 3: Variation of Risk R(T*,T) with standard deviation

The discrimination of target T with T, case-1: R (T,T), is a special case of R (T*,T) at $\sigma$=0°. It represents discrimination of a target with itself, determined using the static RCS, and the value of Risk=0dB is ideal indicating matched targets. The other points in the graph correspond to case-2: R (T*,T) where, the discrimination results are obtained using the dynamic RCS modelled according to the three distributions. The use of dynamic RCS has evidently affected the discrimination results. In other words, even when a target is discriminated with itself, the discrimination value Risk is not '0' and is modified. The Gaussian model exhibits the least variation in discrimination even for large deviations of up to 30°. In the case of the lognormal and Weibull models, the variation in Risk is moderate up to 5° deviation, and beyond 5°, the variation is large exhibiting more than 1dB change.

The discrimination results of case 3: R (Tmv,T) and case 4: R (Tmv*,T) are presented in Figs. 4, 5 and 6 for the three distribution models separately. In Figs 4-6, the Risk value obtained at σ = 0°, correspond to case -1 and case-3 results which are obtained using static RCS.

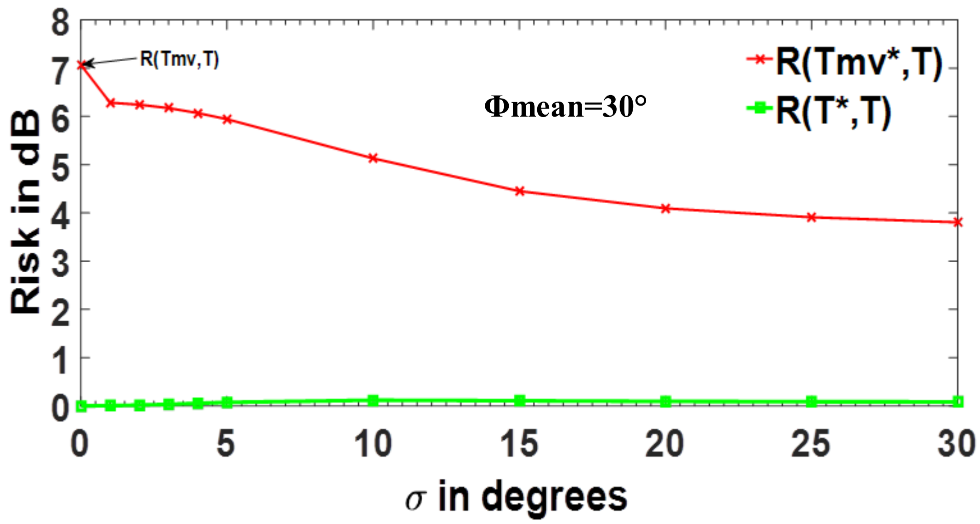


Figure.4.Variation of Risk R (Tmv*,T) and R(T*,T) with dynamic RCS modelled using the Gaussian distribution

In order to determine, the maximum allowable deviation within which the results are acceptable while discriminating a different target, consider Fig. 4. The discrimination margin between the target T and its minor variant Tmv at 30 degree viewing angle (using static RCS) is found to be 7dB (case 3). The discrimination or Risk reduces with increasing standard deviation in aspect angle. Applying the criterion of 3dB threshold for difference in risk, for real discrimination, it is seen from Fig. 4 that the maximum allowed deviation is more than 30° for a Gaussian distribution modelled RCS.

Similar results were obtained using the lognormal and Weibull distributions and are presented in Fig 5 and 6 respectively.

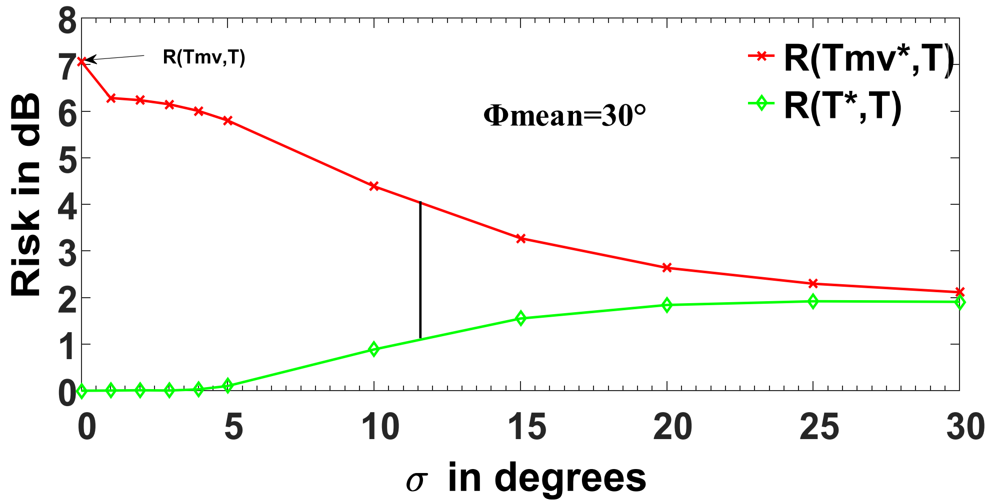


Figure 5. Variation of Risk R (Tmv*,T) and R(T*,T) with dynamic RCS modelled using the lognormal distribution

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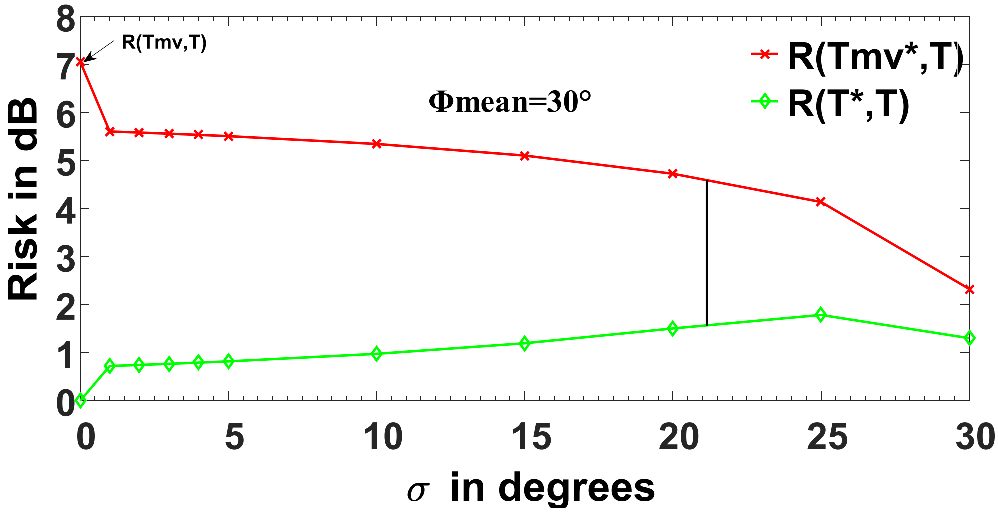


Figure 6. Variation of Risk R (Tmv*,T) and R(T*,T) with dynamic RCS modelled using the Weibull distribution

A vertical line is marked in Fig.5 and 6 which is the 3dB threshold point. Discrimination results obtained below the threshold are acceptable. It can be seen that the maximum allowed deviation is 11.5° and 21° for RCS modelled using lognormal and Weibull distributions respectively. Beyond the threshold point, the discrimination result may not necessarily correspond to the structural variation between the two targets.

## IV. Conclusion

In this study, a method to analyse the effect of dynamic RCS on discrimination of targets is considered. The dynamic RCS was modelled using the Gaussian, lognormal and Weibull probability distributions. Among the three models used, the discrimination obtained using the Gaussian model showed the least variation in discrimination performance even for a standard deviation of 30°. The lognormal and Weibull distribution models show that the acceptable values of discrimination is reduced to deviations of less than 11.5° and 21° respectively. The practical limits of variation in azimuth for most aircraft would be lower than these limits. This leads to the conclusion that the target discrimination technique proposed earlier by the authors is robust and applicable in practical situations.

This study has considered only spatial fluctuation along azimuth direction for modelling dynamic RCS. For the sake of completeness the variations in the pitch of the aircraft should be considered. It is expected that the effect of pitch variation will have similar variations as that of variations in azimuth.

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