Radiation Pattern Analysis and Synthesis of Antenna Arrays using Convex Optimization

Gurdeep Mohal1, Jaspreet Kaur2, and Amanpreet Kaur3

1Department of Electronics and Communication Engineering, RIMT-MAEC, Mandi Gobindgarh, India, E-mail: Er.gurdeepmohal@gmail.com.
2Department of Electronics and Communication Engineering, BBSBEC Fatehgarh Sahib, India, E-mail: er_jaspreet_kaur@yahoo.co.in
3Department of Electronics and Communication Engineering, Thapar University Patiala, India, E-mail: amanpreet.rajpal@gmail.com

Abstract: Antenna Array patterns can be analyzed and synthesized in a variety of ways and are very important in almost every field of Antenna applications. Antenna Array patterns can be expressed as Convex Optimization problem and can be optimized by a number of methods available nowadays, e.g. Interior Point Methods, Genetic Algorithm, SCHELKUNOFF’S METHOD. These methods usually analyzed the Antenna Patterns in terms of Side Lobe Level, Main Lobe, Noise Power, Signal to Noise ratio. In this paper, we consider a uniform linear Antenna array with 40 elements with two equality constraints. It has been verified through the simulation results, using Convex Optimization that the side lobes and the major lobes and beam width of side lobes can now be optimized with precision as well as visible on the plots.

Keywords: Convex, Uniform antenna, Optimization, Side lobe level, Beam width.

1. INTRODUCTION

A single element Antenna is unable to meet the required gain and radiation pattern. So the solution is, to combine several single element antennas to form an Antenna Array. Antenna arrays are used to direct radiated power in a particular direction. There are many types of Antenna Arrays. In this paper, we are considering only Linear Antenna Arrays. After finding the array response, signals are weighted and summed up to obtain the beam pattern.

Antenna Array Pattern synthesis problem consists of finding the optimum weights to satisfy the given requirements. The 1st synthesis problem has been studied by Schelkunoff [1] or Dolph [2]. Literature is available on antenna array optimization by direct methods Such as genetic algorithms [11] and particle Swarm Optimization [12]. These methods are feasible because of increased computing power. An important comment in [3] is that there is no guarantee to reach at an optimum solution unless the problem is convex. But as stated in [4] not all antenna array problems are convex.

A Linear Convex Optimization problem can be solved by many available optimization algorithms. Even Nonlinear Convex Optimization problems can be solved by using Karmarkars Interior Point method [5].

In this paper, we formulate the importance of Convex Optimization in synthesis and analysis of Linear Antenna arrays and show that the side lobe level can be minimized by Convex Optimization.

2. PRESENT WORK

2.1 Linear Array

Consider a linear array of n isotropic elements of equal amplitude and spacing d as shown in figure. The total electric field E in direction ø is given by

\[ E = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \ldots + e^{jn-1}\psi \]

Where \( \psi \) is the total phase difference of fields from adjacent sources and is given by:

\[ \psi = 2\pi (d/\lambda) \frac{\cos \phi + \alpha}{\lambda/2} \]

where \( \phi \) is the phase difference between the feed currents of adjacent sources. \( \phi \) will be same for all the sources.

In case of linear arrays, elements are equally spaced at \( \lambda/2 \) distance.
The array factor of this linear antenna depends upon the number of elements, the element spacing, amplitude, and phase applied to each element.

Array factor is given by

\[ A = \sum_{i=1}^{N} \exp \left( j2\pi / \lambda \left( x_i \cos \phi + y_i \sin \phi \right) \right) \]

The \( N \) signal outputs are converted to complex numbers weighted by weights \( Wi \) and summed up to give the linear array pattern

\[ H(\phi) = \sum_{i=1}^{N} w_i \exp \left( j2\pi / \lambda \left( x_i \cos \phi + y_i \sin \phi \right) \right) \]

where \( w_i = [w_1, w_2, w_3, \ldots, w_N]^T \)

is the complex weight vector to be designed.

### 2.2 Beam Width or HPBW of Antenna

Beam width is a measure of directivity. It is angular width in degrees, measured on the radiation pattern between points where the radiated power has fallen to half its maximum value called Half Power Beam Width or the Gain is one half the maximum value. Beam width is the product of individual antenna pattern and array factor shape.

For reflector antenna Beam width is given by

\[ \text{HPBW} = \alpha = k\lambda / D \]

\( K \) is the factor that depends on the shape of the reflector and the method of illumination. For a typical antenna \( k = 70^\circ \).

Angular width between the first nulls or first side lobes is called Beam Width between the First Nulls (BWFN).

The half power levels occurs at those angles for which

\[ E(\theta, \phi) = \frac{1}{\sqrt{2}} = 0.707 \]

Gain may be expressed in terms of Beam width

\[ G = \eta (\Pi K / \alpha)^2 \]

### 2.3 Convex Optimization

In Mathematics, the term Optimization refers to the study of problems in which we minimizes or maximizes a real function by choosing the real values from allowed set. A mathematical optimization problem has the form:

\[ \text{minimize } f_0(x) \]

subject to \[ f_i(x) < b_i \quad i = 1, \ldots, m \]

Here vector \( x \) is the optimization variable of the problem, the function \( f_0 : \mathbb{R}^n \to \mathbb{R} \) is the objective function, the function \( f_i : \mathbb{R}^n \to \mathbb{R} \quad i = 1, \ldots, m \) are constrained functions.

A Convex Optimization problem is a problem where all of the constraints are convex functions and the objective is a convex function if minimizing or concave if maximizing. Linear functions are convex that’s why we are using convex optimization.

The following are the analysis about the Convex Optimization problem:

- if a local minima exists, then is a global minima.
- the set of all global minima is a convex.
- if the function is strictly convex, then there exists at most one minimum.

The various Convex Optimization problems are: Least square, linear programming, Conic optimization, Geometric programming, Semi definite programming, etc.

Convex optimization problems can be solved with a variety of methods like Ellipsoid method, Sub gradient method, cutting plane method, Interior-point method.

### 2.4 Interior Point Method

In very large-scale optimization problems, it is not possible to form Jacobian and Hessian matrices. In that case, interior point method is used. Actually Interior Point method is very much efficient in obtaining the optimal solution of non linear equations. There are a number of interior point methods like: Barrier method, Inequality constrained minimization, Logarithmic barrier method, Primal interior point method .These methods gained great popularity after Karmarkar [6] given these methods for polynomial. A large amount of work has been done on this, given by Gonzage [7] and Nesterov [8] who developed a very general framework for solving nonlinear convex optimization problems using IPM.

Interior point method moves in the interior of the feasible region hoping to bypass many corner points on the boundary of the region.

### 2.5 Bisection Method

Bisection method is a practical method to find the roots of an equation. It repeatedly bisects an interval then selects a subinterval in which a root must lie for further processing. The method is applicable when we wish to solve the equation \( f(x) = 0 \) for the scalar variable \( x \), where \( f \) is a continuous function.

Suppose that \( c = f(a) < 0 \) and \( d = f(b) > 0 \). If \( f \) is continuous then it must be zero at some \( x \) between \( a \) and \( b \). The bisection method then consists of looking half way between \( a \) and \( b \) for zero of \( f \), i.e. let \( x = \frac{a + b}{2} \) and evaluate \( y = f(x) \).

Unless this is zero, then from signs of \( c, d, y \) we can decide which new interval to subdivide. This is done by flow control in most common way of if………else…….end statement.
3. RESULT AND DISCUSSION

3.1 Problem Statement

We have to minimize the beam pattern level given by (1) with the possible constraints

\[
\min \max_{i=1,2,...,N} H(\phi_i)
\]

subject to \( H(\phi_o) = 1 \)

The formulation of Convex Optimization problem is given by Lasdon [9]. Now to apply the Interior point method, we have to eliminate the equality constraint first by expressing the last weight with the first one

\[
w_N = e^{(j2\pi/\lambda)x_N \cos(\phi_o)}
\]

\[
(1 - \sum_{i=1}^{N-1} w_i e^{-(j2\pi/\lambda)x_i \cos(\phi_i)})
\]

Now replace the objective function with a new variable \( p \) by adding new constraints.

Therefore it is easy to express the original problem in the form of convex problem which can be easily solved with interior point method.

Further we have to find out the feasible solutions for beam width of array. This can be done by Bisection method

3.2 Synthesis Results

3.2.1 Linear Array Analysis

As given in the problem statement, we worked to optimize a linear antenna array. The array has 40 elements with inter element spacing of 0.45\( \lambda \). The main lobe direction is 60° with unit sensitivity.

The original diagram without optimization is given in Fig. 1 (a).

Figure 1 (b) gives the optimal pattern which minimizes the side lobe level between 0 and 180 degrees. The main lobe shape and position is exact as without optimization.

We have taken only two constraints so the result is not satisfied. With some new constraints we can find the more optimal solution.

In Fig. (1), the main beam direction is 60 degree and the width of the main beam is 50 to 70 degrees. We have taken the nulls at five different points shown by pink lines. The plot shows the various side lobes with unequal amplitude at various positions.

The main beam is sharpened as the no of elements increases.

3.2.2 Beam Width Analysis

In this paper we have shown that, antenna gain is degraded if a low side lobe level is sought, so that expansion of antenna beam width is evitable. In this paper we have used weighting with window functions to achieve maximum gain for desired beam width.

Figure 2 shows the minimized beam width pattern of an antenna array with 36 elements and the target direction for the main lobe is 60 degrees. In this example we have taken the minimum side lobe level of \(-20\) dB and maximum half beam width is 60.

We use the bisection algorithm to find out the feasible value for beam width which comes out to be 12 degrees.

The reference side lobe level we have taken is \(-20\) dB.

By adjusting the phase of central element we can increase the directivity and decrease the beam width as well as minimize the side lobe level.
### 4. CONCLUSION

We have shown that how Convex Optimization can be used to design Optimal Antenna Arrays. We have used interior point method which finds the global optimum values with precision. So, Convex Optimization is an excellent tool for pattern synthesis. We have a number of convex optimization algorithms but still, a lot of work can be done in this field. We have shown in this paper that Convex Optimization can solve linear as well as non-linear problems and we have chosen simple numerical examples. This can be used in almost every kind of application of optimization.

We have taken the SLL and antenna beam width, one can optimize the pattern in terms of other antenna parameters.

### REFERENCES


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without optimization</th>
<th>With optimization</th>
<th>Method used for Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side lobe level</td>
<td>– 8.91dB</td>
<td>– 25.3 dB</td>
<td>Optimization is done using matlab</td>
</tr>
<tr>
<td>Beam width</td>
<td>8 degrees</td>
<td>12 degrees</td>
<td>Optimization is done using bisection method and reference SLL taken is –20 dB</td>
</tr>
</tbody>
</table>

![Fig. 2: Optimized Beam Width Array Pattern]