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A Novel Construction Technique for Design of Generalized Orthogonal Codes to Wireless Communication

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Abstract: *The theory of orthogonal designs dates back over a century. Since Radon's classical result implying the set of dimensions for which real square orthogonal designs exist, several generalizations of real square orthogonal designs have followed, including generalized real orthogonal designs and complex orthogonal designs, generalized complex orthogonal designs, and generalized complex linear processing orthogonal designs. Tarokh, Jafarkhani and Calderbank pioneered using generalized complex orthogonal designs to construct space-time Block codes (STBCs), which are used to transmit data over wireless channels using multiple transmit antennas. Their work extends Alamouti's scheme for wireless communications with two transmit antennas. In this work a construction technique for generalized Complex linear processing orthogonal designs, is introduced which are p × n matrices X Satisfying $X^H X = fI$, where f is a complex quadratic form, I is the identity matrix, and X has complex entries. These matrices generalize the familiar notations of orthogonal designs and generalized complex orthogonal designs. We explain the application of these matrices to space-time block coding for multiple-antenna wireless communications. In particular, the Practical strengths of the space-time block codes constructed using the proposed technique (**i.e. Quasi Orthogonal Space time block codes**) are discussed and Interference can also be eliminated.*

Keywords: *Antenna Diversity, Rayleigh Fading, Space–time Coding, Transmit Diversity, Interference.*

1. INTRODUCTION

Multiple antenna systems have been of great interest in recent times because of their ability to support higher data rates at the same bandwidth and noise conditions. For two transmit antennas, Altamonte's orthogonal design gave a full-rate space-time block code with full diversity. More general orthogonal designs were later proposed by Tarokh et al. and Tirkkonen that had simple single symbol decoders while offering full diversity. Recently, complex orthogonal designs with maximal rates have been proposed by Liang where the entries are restricted to be the complex modulated symbols or their conjugates with or without a sign change. The upper bounds of the rates of generalized complex orthogonal space-time block codes were given in one of the key aspects of orthogonal designs has been to ensure diversity for *any* symbol constellation. For more than two transmit antennas and complex constellations, these codes offered on the average a rate of less than one symbol per channel use, where each symbol time period corresponds to a channel use. The highest theoretical code rate for full diversity code when the symbols are constrained to be chosen from the *same* constellation was shown to be one symbol per channel use. This constraint is relaxed by using rotated constellations and indeed many of the recent papers give space-time codes that offer full diversity for more than one symbol per channel.

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More recently, a different approach has been attempted to yield the full diversity where the notion of diversity is made specific to a constellation, and this is also referred to as modulation diversity. More specifically, it has been shown that full-rate and full-modulation diversity is achievable with constellation rotation or linear constellation precoding, where the transmitted signal is a multiplication of a unitary matrix with a diagonal matrix whose diagonal elements are a function of linearly precoded (or rotated) information symbols. This makes the test of full diversity or the rank criterion trivial by ensuring with proper precoding or constellation rotation that no element in the diagonal becomes zero while taking the difference of two distinct codewords. A similar idea has been presented before in for rotated binary phase shift keying (BPSK) modulation.

The issue of smaller code rate (less than one symbol per channel use) for complex orthogonal designs has been addressed in recent times by the design of quasi-orthogonal codes for achieving higher data rates The quasi-orthogonal codes were given for 4 transmit antennas with rate 1, and 8 transmit antennas with rate 3/4. These codes sacrificed some orthogonality by making *subsets* of symbols orthogonal to each other instead of making every single symbol orthogonal to any other. Because of this relaxation of constraints, these codes achieve higher code rates that were hitherto not possible with orthogonal codes. It was shown that performance of the above quasi orthogonal codes can be improved with constellation rotation. Constellation rotation has also been discussed as a technique to improve the performance of space-time block codes.

In this thesis, we build on earlier work on orthogonal designs and achieving modulation diversity by constellation rotation to propose a quasi-orthogonal structure to iteratively construct full-diversity space-time codes for *any* transmit antennas. These codes have half the symbols orthogonal to the other half, which allows each orthogonal half to be decoded separately without any loss of performance. Hence the decoding complexity of such a code is considerably smaller. We show that these codes achieve full diversity with appropriate constellation rotations. If the transmit antennas are a power of 2, then these codes are also delay "optimal," that is, the length of block code in symbol periods is same as the number of transmit antennas.

2. ENCODING OF QOSTBC

The encoding using N transmits antennas and a space–time block code is described t. Let $\mathbb{C}$ denote a signal constellation of size 2^b. At time one; *Kb* bits arrive at the encoder, where K is the number of the variables in the transmission matrix. These *Kb* bits choose *K* constellation symbols $s_1, s_2, ..., s_k$. The encoder replaces s_k everywhere for x_k in the transmission matrix for all $1 \le k \le K$. Let us denote the resulting matrix *C*. Then at time t, $t = 1, 2, ...\ N$ the nth element of the tth row *C* of C_{tn}, is transmitted using antennas $n = 1, 2, ...N$. We emphasize that all these transmissions are simultaneous and that all the transmitted signals have the same time duration. Since elements of $\mathbb{C}$ are linear combinations of $x_1, x_2, ..., x_k$ and their conjugates, the encoding only requires linear processing.

3. TRANSMISSION MODEL

We consider a wireless communication system with *N* antennas at the base station and *M* antennas at the remote. The channel is assumed to be a flat fading channel and the path gain from transmit antenna *n* to receive antenna *m* is defined to be $\alpha_{n,m}$. The path gains are modeled as samples of independent complex Gaussian random variables. The real part and imaginary part of path gain have equal variance 0.5. This assumption can be relaxed without any change to the method of encoding and decoding. The wireless channel is assumed to be quasi-static so that the path gains are constant over a frame of length *T* and vary from one frame to another. At time *t*, the received signal $r_{t,m}$ at antenna *m* is given by

$$t_{t,m} = \sum_{n=1}^{N} \alpha_{n,m} C_{tn} + \eta_{t,m} \quad (1)$$

where the noise samples $\eta_{t,m}$ are independent samples of a zero-mean complex Gaussian random variable. The real part and imaginary part of noise have equal variance *N*/(2 SNR). The average energy of the symbols transmitted from each antenna is normalized to be 1, so that the average power of the received signal at each receive antenna *N* is and the signal-to-noise ratio is SNR.

4. PAIR WISE DECODING

Full rate orthogonal designs with complex elements in its transmission matrix are impossible for more than two transmit antennas. The only example of a full-rate full diversity complex space-time block code using orthogonal designs is the Alamouti scheme. Here we rewrite the generator matrix for the Alamouti code to emphasize the indeterminate variables s_1 and s_2 in the design:

$$\mathbb{C}(s_1, s_2) = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \quad (2)$$

The main properties of an orthogonal design are simple separate decoding and full diversity. To design full rate codes, we relax the simple separate decoding property. So we consider codes for which decoding pairs of symbols independently is possible. We call this class of codes Quasi-Orthogonal space-time block codes(QOSTBCs).

First, let us consider the following QOSTBC

$$\mathbb{C} = \begin{pmatrix} \mathbb{C}(s_1, s_2) & \mathbb{C}(s_3, s_4) \\ -\mathbb{C}(s_3, s_4) & \mathbb{C}(s_1, s_2) \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{pmatrix} \quad (3)$$

In this design each column of generator matrix is orthogonal to all other column except one. As a result a pair wise decoding of the symbols is possible sing rotation for some of the symbols, full diversity QOSTBCs are realizable. We decoding when there is one user. The multiuser case is next to be studied. Assuming perfect channel state information is available; the receiver computes the decision metric

$$\sum_{m=1}^{M} \sum_{t=1}^{4} \left| r_{t,m} = \sum_{n=1}^{4} \alpha_{n,m}(1) \zeta_{tn}(1) \right|^2 \quad (4)$$

Over all possible symbols to replace s_1, ..., s_4 in C and decides in favour of constellation symbols that minimize the sum. Since we have only one user and for simplicity specify one receiver antenna, we do not mention indexing of group of or receive antenna. Simple algebraic manipulation shows that ML decoding.

We showed that this metric is the sum of K components each involving only the variable x_k, $k = 1, 2, ...K$ indeed, if the metric (4) is expanded, the cross terms involving $\alpha_{p,m}\alpha^*_{q,m}$, $1 \le p \ne q \le N$ are canceled out since pth and qth columns of ζ are orthogonal to each other. Thus the sum has *K* components involving only the variable x_k, $k = 1, 2, ...K$,. It can be further proved that each component can be computed using only linear processing. From eq. 3, it is seen that the minimum rank of matrix $C(x_1 - \tilde{x}_1,\ x_2 - \tilde{x}_2,\ x_3 - \tilde{x}_3,\ x_4 - \tilde{x}_4)$, the matrix constructed from by replacing from C by replacing s_i from $x_i - \tilde{x}_i$, is 2. Therefore, a diversity of 2M is achieved while the rate of the code is one. Note that it has been proved that the maximum diversity of 4M for a rate one code is impossible in this case. Now, if we define $v_i, i = 1,2,3,4$ as the ith column of C, it is easy to see that

$$(v_1, v_2) = (v_1, v_3) = (v_2, v_4) = (v_3, v_4) = 0 \tag{5}$$

Where $\langle v_i, v_j \rangle = \sum_{l=1}^{4} (v_i)_l (v_j)_{l*}$ is the inner product of vectors v_i and v_j. Therefore, the subspace created by v_1 and v_4 is orthogonal to the subspace created by v_2 and v_3. Using this orthogonality, the maximum-likelihood decision metric (4) can be calculated as the sum of the two terms $f_{13}(s_1,s_3) + f_{24}(s_2,s_4)$, where f_{13} is independent of s_2 and s_4 and f_{24} is independent of s_1 and s_3. Thus, the minimization of (3) is equivalent to minimizing these two terms independently. In other words first the decoder finds the pair (s_1, s_3) that minimizes the $f_{13}(s_1, s_3)$ among all possible (s_1, s_3) pairs. Then, or in parallel, the decoder selects the pair (s_2, s_4) which minimizes the $f_{24}(s_2, s_4)$. This reduces the complexity of decoding without sacrificing the performance. The pairs (s_1, s_3) and (s_2, s_4) can be decoded separately and the scheme is pair wise decidable.

In this a new decoding method was introduced by which differential decoding of a QOSTBC for non-coherent systems became possible. Let us review this decoding method. For $M = 1$ receive antenna, let us define the received signals at four time slots by r_1, r_2, r_3, r_4 then, the set of input-output equations is, Simple manipulation of (4) provides the following formulas for f_{13} (.) and f_{24}(.):

$$\begin{aligned} f_{13}(s_1,s_3) = \sum_{m=1}^{4} & \left(|s_1|^2 + |s_3|^2\right)\left(\sum_{n=1}^{4}|\alpha_{n,m}|^2\right) \\ & 2\Re(-\alpha_{1,m} r_{1,m}^* - \alpha_{2,m}^* r_{2,m} - \alpha_{3,m}^* r_{3,m} - \alpha_{4,m} r_{4,m}^*)s_1 \\ & +(-\alpha_{4,m} r_{1,m}^* + \alpha_{3,m}^* r_{2,m} + \alpha_{2,m}^* r_{3,m} - \alpha_{1,m} r_{4,m}^*)s_3\} \\ & 4\Re\{\alpha_{1,m}\alpha_{4,m}^* - \alpha_{2,m}^*\alpha_{3,m}\}\Re\{s_1 s_3^*\}] \end{aligned} \tag{6}$$

$$\begin{aligned} f_{2,4}(s_2,s_4) = \sum_{m=1}^{4} & \left[\left(|s_2|^2 + |s_4|^2\right)\left(\sum_{n=1}^{4}|\alpha_{n,m}|^2\right) + \right. \\ & 2\Re(-\alpha_{2,m} r_{1,m}^* - \alpha_{1,m}^* r_{2,m} - \alpha_{4,m}^* r_{3,m} - \alpha_{3,m} r_{4,m}^*)s_2 \\ & +(-\alpha_{3,m} r_{1,m}^* + \alpha_{4,m}^* r_{2,m} + \alpha_{1,m}^* r_{3,m} - \alpha_{2,m} r_{4,m}^*)s_4\} \\ & +4\Re\{\alpha_{2,m}\alpha_{3,m}^* - \alpha_{1,m}^*\alpha_{4,m}\}\Re\{s_2 s_4^*\} \end{aligned} \tag{7}$$

$$R_1 = S_1 H_1 + N_1 \tag{8}$$

Where

$$R_1 = (r_{13,1}, r_{24,1})^T = (r_1 + r_3, r_2 + r_4)^T$$

$$H_1 = (\alpha_{13,1}, \alpha_{24,1})^T = (\alpha_1 + \alpha_3, \alpha_2 + \alpha_4)^T$$

$$N_1 = (\eta_1 + \eta_3, \eta_2 + \eta_4)^T$$

$$S_1 = \begin{pmatrix} s_{13,1} & s_{24,1} \\ -s_{24,1}^* & s_{13,1}^* \end{pmatrix} = \begin{pmatrix} s_1 + s_3 & s_2 + s_4 \\ -(s_2^* + s_4^*) & s_1^* + s_3^* \end{pmatrix}$$

And

$$R_2 = S_2 H_2 + N_2 \tag{9}$$

Where

$$R_2 = (r_{13,2}, r_{24,2})^T = (r_1 - r_3, r_2 - r_4)^T$$

$$H_2 = (\alpha_{13,2}, \alpha_{24,2})^T = (\alpha_1 - \alpha_3, \alpha_2 - \alpha_4)^T$$

$$N_2 = (\eta_1 - \eta_3, \eta_2 - \eta_4)^T$$

$$S_1 = \begin{pmatrix} s_{13,2} & s_{24,2} \\ -s_{24,2}^* & s_{13,2}^* \end{pmatrix} = \begin{pmatrix} s_1 - s_3 & s_2 - s_4 \\ -(s_2^* - s_4^*) & s_1^* - s_3^* \end{pmatrix}$$

We consider (8) and (9) as two equivalent subsystems, each of which has two transmit antennas. We can see that and have the structure of Alamouti code. This is the key property in multi-user decoding .The ML decoding metric for this system is

$$\begin{aligned} X = & |r_{13,1} - \alpha_{13,1}s_{13,1} - \alpha_{24,1}s_{24,1}|^2 + \\ & |r_{24,1} + \alpha_{13,1}s_{24,1}^* - \alpha_{24,1}s_{13,1}|^2 \\ & + |r_{13,2} - \alpha_{13,2}s_{13,2} - \alpha_{24,2}s_{24,2}|^2 + \\ & |r_{24,2} + \alpha_{13,2}s_{24,2}^* - \alpha_{24,2}s_{13,2}^*|^2 \end{aligned} \tag{10}$$

We need to find the symbols s_1,s_2,s_3,s_4 that minimizes X. Expanding the expression for we get

$$X = |r_1|^2 + |r_2|^2 + |r_3|^2 + |r_4|^2 + 2f_{13}(s_1, s_3) + f_{24}(s_2, s_4) \tag{11}$$

Therefore, the choice of $\{s_1,s_2,s_3,s_4\}$ that minimize X, will minimize the ML metric of the original system as well. In other words, our transformation is lossless, and the Error performances is same as the optimal decoding of the system in (3). For multi-user detection of Alamouti equipped transmitters, we decode the whole group. Users are sending QOSTBCs, at each receive antenna m we receive the following signals in the four time slots

$$\begin{aligned} r_{1,m} &= \sum_{j=1}^{J} \alpha_{1,m}(j)s_1(j) + \alpha_{2,m}(j)s_2(j) + \alpha_{3,m}(j)s_3(j) + \alpha_{4,m}(j)s_4(j) + \eta_{1,m} \\ r_{2,m} &= \sum_{j=1}^{J} -\alpha_{1,m}(j)s_2^*(j) + \alpha_{2,m}(j)s_1^*(j) - \alpha_{3,m}(j)s_4^*(j) + \alpha_{4,m}(j)s_3^*(j) + \eta_{2,m} \\ r_{3,m} &= \sum_{j=1}^{J} \alpha_{1,m}(j)s_3(j) + \alpha_{2,m}(j)s_4(j) + \alpha_{3,m}(j)s_1(j) + \alpha_{4,m}(j)s_2(j) + \eta_{3,m} \\ r_{4,m} &= \sum_{j=1}^{J} -\alpha_{1,m}(j)s_4^*(j) + \alpha_{2,m}(j)s_3^*(j) - \alpha_{3,m}(j)s_2^*(j) + \alpha_{4,m}(j)s_1^*(j) + \eta_{4,m} \end{aligned} \tag{12}$$

Where $\eta_{i,m}s$ are i.i.d zero mean Gaussian random variables with variance $\frac{4}{SNR}$, similarly, for the signals at each receive antenna we form two equivalent 2-antennas. System follows

$$R_{1,m}=\begin{pmatrix} r_{1,m}+r_{3,m} \\ r_{2,m}+r_{4,m} \end{pmatrix}=\sum_{j=1}^{J}\begin{pmatrix} s_1(j)+s_3(j) & s_2(j)+s_4(j) \\ -\left(s_2(j)+s_4(j))^*\right) & s_1(j)+s_3(j))^* \end{pmatrix}$$
$$\begin{pmatrix} \alpha_{1,m}(1)+\alpha_{3,m}(1) \\ \alpha_{2,m}(1)+\alpha_{4,m}(1) \end{pmatrix}+\begin{pmatrix} \eta_{1,m}+\eta_{3,m} \\ \eta_{2,m}+\eta_{4,m} \end{pmatrix}$$
$$R_{2,m}=\begin{pmatrix} r_{1,m}-r_{3,m} \\ r_{2,m}-r_{4,m} \end{pmatrix}=\sum_{j=1}^{J}\begin{pmatrix} s_1(j)-s_3(j) & s_2(j)-s_4(j) \\ -\left(s_2(j)-s_4(j))^*\right) & s_1(j)-s_3(j))^* \end{pmatrix}$$
$$\begin{pmatrix} \alpha_{1,m}(1)-\alpha_{3,m}(1) \\ \alpha_{2,m}(1)-\alpha_{4,m}(1) \end{pmatrix}+\begin{pmatrix} \eta_{1,m}-\eta_{3,m} \\ \eta_{2,m}-\eta_{4,m} \end{pmatrix}$$

Without loss of generality we assume that we eliminate the effects of user No.1 first. By applying complex conjugation on the second row of both systems we get

$$R_{1,m}^{T}=\begin{pmatrix} r_{1,m}+r_{3,m} \\ r_{2,m}^{*}+r_{4,m}^{*} \end{pmatrix}^{T}=\sum_{j=1}^{J}\left(s_1(j)+s_3(j)s_2(j)+s_4(j)\right)$$
$$\begin{pmatrix} \alpha_{1,m}(j)+\alpha_{3,m}(j) & \left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^{*} \\ \alpha_{2,m}(j)+\alpha_{4,m}(j) & -\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^{*} \end{pmatrix}$$
$$+\begin{pmatrix} \eta_{1,m}+\eta_{3,m} \\ \eta_{2,m}+\eta_{4,m} \end{pmatrix}^{T}$$
$$=\sum_{j=1}^{J}\left(s_1(j)+s_3(j),s_2(j)+s_4(j)\right)\cdot\zeta$$
$$\left(\alpha_{1,m}(j)+\alpha_{3,m}(j),\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)+N_{1,m} \tag{13}$$

and

$$R_{2,m}^{T}=\begin{pmatrix} r_{1,m}-r_{3,m} \\ r_{2,m}^{*}-r_{4,m}^{*} \end{pmatrix}^{T}=\sum_{j=1}^{J}\left(s_1(j)-s_3(j),s_2(j)-s_4(j)\right).$$
$$\begin{pmatrix} \alpha_{1,m}(j)-\alpha_{3,m}(j) & \left(\alpha_{2,m}(j)-\alpha_{4,m}(j)\right)^{*} \\ \alpha_{2,m}(j)-\alpha_{4,m}(j) & -\left(\alpha_{2,m}(j)-\alpha_{4,m}(j)\right)^{*} \end{pmatrix}$$
$$+\begin{pmatrix} \eta_{1,m}-\eta_{3,m} \\ \eta_{2,m}-\eta_{4,m} \end{pmatrix}^{T}$$
$$=\sum_{j=1}^{J}\left(s_1(j)-s_3(j),s_2(j)-s_4(j)\right)\cdot\zeta$$
$$\left(\alpha_{1,m}(j)-\alpha_{3,m}(j),\alpha_{2,m}(j)-\alpha_{4,m}(j)\right)+N_{1,m} \tag{14}$$

Where
$$\zeta(x,y)=\begin{pmatrix} x & y^{*} \\ y & -x^{*} \end{pmatrix} \tag{15}$$

The advantage of representing the received signals in the above format is hidden in the structure of the equivalent channel matrices:

$$\Omega_{1,m}(j)=\begin{pmatrix} \alpha_{1,m}(j)+\alpha_{3,m}(j) & \left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^{*} \\ \alpha_{2,m}(j)+\alpha_{4,m}(j) & -\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)^{*} \end{pmatrix}=$$
$$\zeta(\alpha_{1,m}(j)+\alpha_{3,m}(j),\alpha_{2,m}(j)+\alpha_{4,m}(j))$$
$$\Omega_{2,m}(j)=\begin{pmatrix} \alpha_{1,m}(j)-\alpha_{3,m}(j) & \left(\alpha_{2,m}(j)-\alpha_{4,m}(j)\right)^{*} \\ \alpha_{2,m}(j)-\alpha_{4,m}(j) & -\left(\alpha_{1,m}(j)-\alpha_{3,m}(j)\right)^{*} \end{pmatrix}=$$
$$\zeta(\alpha_{1,m}(j)-\alpha_{3,m}(j),\alpha_{2,m}(j)-\alpha_{4,m}(j))$$

Since both $\Omega_{1,m}(j)$ and $\Omega_{2,m}(j)$ are multiple of a unitary matrix, we can simply separate and therefore eliminate the effect of each group form the received signal.

$$R_{1,m}^{'T}\cdot\Omega_{1,m}^{\dagger}(1)=\left(\left|\alpha_{1,m}(1)+\alpha_{3,m}(1)\right|^{2}+\left|\alpha_{2,m}(1)-\alpha_{4,m}(1)\right|^{2}\right)$$
$$+(s_1(j)+s_3(j),s_2(j)+s_4(j))+$$
$$\sum_{j=1}^{J}\left(s_1(j)+s_3(j),s_2(j)+s_4(j)\right).$$
$$\zeta'\left(\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)^{*}+\right.$$
$$\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^{*}\left(\alpha_{2,m}(j)+\alpha_{4,m}\right)),$$
$$(\alpha_{1,m}(j)+\alpha_{3,m}(j))(\alpha_{2,m}(j)+\alpha_{4,m}(j))^{*}$$
$$-\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^{*}\left(\alpha_{1,m}(j)+\alpha_{3,m}(j))\right)+N_{1,m} \tag{16}$$

$$R_{2,m}^{'T}\cdot\Omega_{2,m}^{\dagger}(1)=\left(\left|\alpha_{1,m}(1)-\alpha_{3,m}(1)\right|^{2}+\left|\alpha_{2,m}(1)-\alpha_{4,m}(1)\right|^{2}\right)$$
$$(s_1(j)-s_3(j),s_2(j)-s_4(j))+$$
$$\sum_{j=2}^{J}\left(s_1(j)-s_3(j),s_2(j)-s_4(j)\right).$$
$$\zeta'\left(\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)^{*}+\right.$$
$$\left(\alpha_{2,m}(j)-\alpha_{4,m}(j)\right)^{*}\left(\alpha_{2,m}(1)+\alpha_{4,m}(1)\right)),$$
$$(\alpha_{1,m}(j)-\alpha_{3,m}(j))(\alpha_{2,m}(j)-\alpha_{4,m}(j))^{*}$$
$$-\left(\alpha_{2,m}(j)-\alpha_{4,m}(j)\right)^{*}\left(\alpha_{1,m}(1)-\alpha_{3,m}(1))\right)+N_{2,m} \tag{17}$$

Where

$$\zeta'(x,y)=\begin{pmatrix} x & y \\ -y* & x* \end{pmatrix} \tag{18}$$

$N'_{1,m}$ and $N'_{2,m}$ are the new noise vectors corresponding to the mth receive antenna. Noting that $\Omega_{i,m}$ matrices are multiples of unitary, the distribution of each element of $N'_{1,m}$ will be Gaussian with variance $\frac{8\left(\left|\alpha_{1,m}(1)+\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)+\alpha_{4,m}(1)\right|^2\right)}{SNR}$. The two elements will be still i.i.d. Same thing is true for $N'_{2,m}$ with a change of sign.

Let us consider for all receive antennas $m = 1, 2 ..., J + r - 1$. If we subtract the expression for the receive antenna, $m = 1$, from that of the other antennas we will have $J + r - 2$ equations as follows:

$$\frac{R'^{T}_{1,m}\cdot\Omega^{\dagger}_{1,m}(1)}{\left|\alpha_{1,m}(1)+\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)+\alpha_{4,m}(1)\right|^2}-\frac{R'^{T}_{1,1}\cdot\Omega^{\dagger}_{1,1}(1)}{\left|\alpha_{1,1}(1)+\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)+\alpha_{4,1}(1)\right|^2}$$
$$=\sum_{j=1}^{J}\left(s_1(j)-s_3(j),s_2(j)+s_4(j)\right)\cdot\zeta'\left(A_{1,m}(j),B_{1,m}(j)\right)+$$
$$\frac{N^{\dagger}_{1,m}}{\left|\alpha_{1,m}(1)+\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)+\alpha_{4,m}(1)\right|^2}-\frac{N^{\dagger}_{2,1}}{\left|\alpha_{1,1}(1)+\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)+\alpha_{4,1}(1)\right|^2} \quad (19)$$

and

$$\frac{R'^{T}_{2,m}\cdot\Omega^{\dagger}_{2,m}(1)}{\left|\alpha_{1,m}(1)-\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)-\alpha_{4,m}(1)\right|^2}-\frac{R'^{T}_{1,1}\cdot\Omega^{\dagger}_{1,1}(1)}{\left|\alpha_{1,1}(1)-\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)-\alpha_{4,1}(1)\right|^2}$$
$$=\sum_{j=2}^{J}\left(s_1(j)-s_3(j),s_2(j)-s_4(j)\right)\cdot\zeta'\left(A_{2,m}(j),B_{2,m}(j)\right)+$$
$$\frac{N'_{2,m}}{\left|\alpha_{1,m}(1)-\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)-\alpha_{4,m}(1)\right|^2}-\frac{N'_{2,1}}{\left|\alpha_{1,1}(1)-\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)-\alpha_{4,1}(1)\right|^2} \quad (20)$$

where

$$A_{1,m}(j)=\frac{\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)\left(\alpha_{1,m}(1)+\alpha_{3,m}(1)\right)^*+\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^*\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)}{\left|\alpha_{1,1}(1)+\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)+\alpha_{4,1}(1)\right|^2}$$
$$-\frac{\left(\alpha_{1,1}(j)+\alpha_{3,1}(j)\right)\left(\alpha_{1,1}(1)+\alpha_{3,1}(1)\right)^*+\left(\alpha_{2,1}(j)+\alpha_{4,1}(j)\right)^*\left(\alpha_{2,1}(j)+\alpha_{4,1}(j)\right)}{\left|\alpha_{1,1}(1)+\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)+\alpha_{4,1}(1)\right|^2} \quad (21)$$

$$B_{1,m}(j)=\frac{\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)\left(\alpha_{2,m}(1)+\alpha_{4,m}(1)\right)^*-\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^*\left(\alpha_{1,m}(1)+\alpha_{3,m}(1)\right)}{\left|\alpha_{1,m}(1)+\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)+\alpha_{4,m}(1)\right|^2}$$
$$-\frac{\left(\alpha_{1,1}(j)+\alpha_{3,1}(j)\right)\left(\alpha_{2,1}(1)+\alpha_{4,1}(1)\right)^*-\left(\alpha_{2,1}(j)+\alpha_{4,1}(j)\right)^*\left(\alpha_{1,1}(1)+\alpha_{3,1}(1)\right)}{\left|\alpha_{1,1}(1)+\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)+\alpha_{4,1}(1)\right|^2}$$

and

$$A_{1m}(j)=\frac{\left(\alpha_{1,m}(j)+\alpha_{3,m}(j)\right)\left(\alpha_{2,m}(1)+\alpha_{4,m}(1)\right)^*-\left(\alpha_{2,m}(j)+\alpha_{4,m}(j)\right)^*\left(\alpha_{1,m}(1)+\alpha_{3,m}(1)\right)}{\left|\alpha_{1,m}(1)+\alpha_{3,m}(1)\right|^2+\left|\alpha_{2,m}(1)+\alpha_{4,m}(1)\right|^2}$$
$$-\frac{\left(\alpha_{1,1}(j)+\alpha_{3,1}(j)\right)\left(\alpha_{2,1}(1)+\alpha_{4,1}(1)\right)^*-\left(\alpha_{2,1}(j)+\alpha_{4,1}(j)\right)^*\left(\alpha_{1,1}(1)+\alpha_{3,1}(1)\right)}{\left|\alpha_{1,1}(1)+\alpha_{3,1}(1)\right|^2+\left|\alpha_{2,1}(1)+\alpha_{4,1}(1)\right|^2}$$

$$B_{2,m}(j) = \frac{\left(\alpha_{1,m}(j) - \alpha_{3,m}(j)\right)\left(\alpha_{2,m}(1) - \alpha_{4,m}(1)\right)^* - \left(\alpha_{2,m}(j) - \alpha_{4,m}(j)\right)^* \left(\alpha_{1,m}(1) - \alpha_{3,m}(1)\right)}{\left|\alpha_{1,m}(1) - \alpha_{3,m}(1)\right|^2 + \left|\alpha_{2,m}(1) - \alpha_{4,m}(1)\right|^2}$$
$$- \frac{\left(\alpha_{1,1}(j) - \alpha_{3,1}(j)\right)\left(\alpha_{2,1}(1) - \alpha_{4,1}(1)\right)^* - \left(\alpha_{2,1}(j) - \alpha_{4,1}(j)\right)^* \left(\alpha_{1,1}(1) - \alpha_{3,1}(1)\right)}{\left|\alpha_{1,1}(1) - \alpha_{3,1}(1)\right|^2 + \left|\alpha_{2,1}(1) - \alpha_{4,1}(1)\right|^2} \qquad (22)$$

We note that ζ' is also multiple of a unitary matrix as ζ. It is as if, in equations (17) we have $J + r - 2$ receive antennas and $J - 1$ user groups. Therefore, we can separate and eliminate user No. 2 similar to user No. 1; except that the number of equivalent receive antennas will be one less, ie $(J + r - 2)$. Eliminating user No. 2 gives us $J + r - 3$ equations with $J - 2$ users. If we continue this procedure, we will have equations and our metric is ML, diversity gain will be equal to $4 \times r$. One should note that better performance is possible, if we use more complex decoding methods. For example, since we break down our code to two Alamouti structure, MMSE-ZF method used for 2 transmit antenna systems can be applied. This method that involves a matrix inversion, will provide a better error rate, compared with our simple ZF method. Also, one can decode users on the order of the strength of their respective channel and improve the performance. The main goal of the project is possible to do multiuser detection for any number of transmit antennas, using very few number of receive antenna. There are other structures which provide behaviors similar to those of (3). A few examples are given below

$$\begin{pmatrix} \mathbb{C}_{12} & \mathbb{C}_{34} \\ -\mathbb{C}_{34} & \mathbb{C}_{12} \end{pmatrix} \quad \begin{pmatrix} \mathbb{C}_{12} & \mathbb{C}_{34} \\ \mathbb{C}_{34} & -\mathbb{C}_{12} \end{pmatrix} \quad \begin{pmatrix} \mathbb{C}_{12} & \mathbb{C}_{34} \\ \mathbb{C}_{34}^* & -\mathbb{C}_{12}^* \end{pmatrix}$$

The main idea in the structure of the transmission matrix $\mathbb{C}$ in (3) is to build a (4×4) matrix from two (2×2) matrices to keep the transmission rate fixed. A similar idea can be used to combine two rate 3/4 transmission matrices (4×4) to build a rate 3/4 transmission matrixes (8×8) and so on. An example of a (8×8) matrix which provides a rate 3/4 code is given below

$$\begin{pmatrix} s_1 & s_2 & s_3 & 0 & s_4 & s_5 & s_6 & 0 \\ -s_2^* & s_1^* & 0 & -s_3 & s_5^* & -s_4^* & 0 & s_6 \\ s_3^* & 0 & -s_1^* & -s_2 & -s_6^* & 0 & s_4^* & s_5 \\ 0 & -s_3^* & s_2^* & -s_1 & 0 & s_6^* & -s_5^* & s_4 \\ -s_4 & -s_5 & -s_6 & 0 & s_1 & s_2 & s_3 & 0 \\ -s_5^* & s_4^* & 0 & s_6 & -s_2^* & s_1^* & 0 & s_3 \\ s_6^* & 0 & -s_4^* & s_5 & s_3^* & 0 & -s_1^* & s_2 \\ 0 & s_6^* & -s_5^* & -s_4 & 0 & s_3^* & -s_2^* & s_1 \end{pmatrix}$$

In this code if we define v_i, $i = 1, 2, 3 \ldots, 8$ as the ith column, we have

$$\langle v_1, v_i \rangle = 0, i \neq 5, \quad \langle v_2, v_i \rangle = 0, i \neq 6,$$
$$\langle v_3, v_i \rangle = 0, i \neq 7, \quad \langle v_4, v_i \rangle = 0, i \neq 8,$$
$$\langle v_5, v_i \rangle = 0, i \neq 1, \quad \langle v_6, v_i \rangle = 0, i \neq 2,$$
$$\langle v_7, v_i \rangle = 0, i \neq 3, \quad \langle v_8, v_i \rangle = 0, i \neq 4,$$

The maximum-likelihood decision metric (3) can be calculated as the sum of three terms $f_{13}(s_1, s_3) + f_{24}(s_2, s_4) + f_{35}(s_3, s_5)$ and similarly the decoding can be done using pairs of constellation symbols.

5. RESULTS AND CONCLUSIONS

In this section, we provide simulation results for the proposed code and compare it with the results for the orthogonal codes presented in. In all simulations, we consider one receive antenna. Since a full-rate full-diversity code exists for real signal constellations, there is no advantage in using the quasi-orthogonal code and binary phase-shift keying that results in the transmission of 1 bits/s / Hz. Figure 1, provides simulation results for the transmission of 2 bits/s / Hz using four transmit antennas using the new rate one quasi-orthogonal, the rate 1 full-diversity orthogonal code and the uncoded 4-QAM. Note that we have used the appropriate modulation schemes to

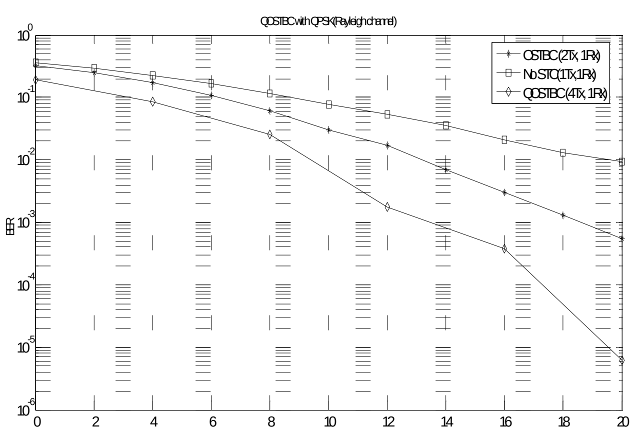

Figure 1: Bit-error Probability versus SNR for Quasi-Orthogonal Space–time Block Codes at 2 bits/s/Hz; 1 Receive Antenna.

provide the desired transmission rate for the space–time block codes, i.e., 4-QAM for the rate one code.

Simulation results show that full transmission rate is more important for very low SNRs and high BERs, while full diversity is the right choice for high SNRs and low BERs.This is due to the fact that the degree of diversity dictates the slope of the BER-SNR curve. Therefore, although a rate one quasi-orthogonal code starts from a better point in the BER-SNR plane, a code with full-diversity benefits more from increasing the SNR. Therefore, the BER-SNR curve of the full-diversity scheme passes the curve for the new code at some moderate SNR. Also, note that the receiver of the full-diversity codes can decode the symbols one by one while the decoding for the new rate one quasi-orthogonal code is done for pairs of symbols. This means that the decoding complexity of the full-diversity orthogonal codes is lower although both codes have a very low decoding complexity. The encoding complexities of the two systems are small and the same. Note that unlike the case of full-diversity orthogonal designs, we have not proved that codes with rates greater than one are impossible for the new structure of quasi-orthogonal space–time block codes.

At BER of with full data rate, QPSK code gives about 11dB gain over the use of an uncoded 4-QAM data transmission. at BER of with full data rate, 4-QAM code gives about 6dB gain over the use of an uncoded 4-QAM data transmission

6. FUTURE WORK

An interesting open problem for future work is the study of the maximum possible rate for a given number of transmits antennas. We conjecture that the rate of a complex code which provides half of the full diversity and works with pairs of symbols cannot be more than one.

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