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COMPARISON OF SINGULAR VALUE DECOMPOSITION, PRINCIPLE COMPONENT ANALYSIS, INDEPENDENT COMPONENT ANALYSIS FOR AN IMAGE

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ABSTRACT

It is well known that the images, often used in variety of computer applications, are difficult to store and transmit. One possible solution to overcome this problem is to use a data compression technique where an image is viewed as a matrix and then the operations are performed on the matrix. Image compression is achieved by using Singular Value Decomposition (SVD) technique on the image matrix. PCA has been widely applied in the area of image compression in various forms. PCA has been applied as standalone image compression technique as well as pre-processing or post-processing step in combination To apply ICA to images, each sample of X usually contains the pixels in an image block. Independent component analysis (ICA) considers a class of probabilistic generative models in which a random vector X is obtained according to X = AS, where A is an NXM unknown mixing matrix and S is a vector of independent sources. The comparative study is conducted with the help of Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), compression ratio.

Keywords: Image compression; SVD, PCA, ICA.

1. INTRODUCTION

Images used in Image Processing application, require big storage space and a considerable bandwidth for transmission. One of the possible solutions to this problem is data compression whose main goal is to reduce the quantity of data used to represent a digitized image. The data compression techniques always eliminate the redundant data. The process of compressing (or decompression) images can be classified into two categories, lossless compression and lossy compression [1]. Lossless methods produce exact copy of the original data and have a limit of reduction in terms of the entropy. Lossy techniques usually work in terms of the incapacity of the human visual system to detect small details and variations in images. Singular Value Decomposition is an effective tool for minimizing data storage and data transfer. One of the possible solutions to this problem is data compression whose main goal is to reduce the quantity of data used to represent a digitized image [2] [6]. The data compression techniques always eliminate the redundant data.

The standard PCA works on 1-dimensional vectors which has inherent problem of dealing with high dimensional vector space data such as images, whereas 2DPCA directly works on matrices i.e. in 2DPCA, PCA technique is applied directly on original image without transforming into 1 dimensional vector. This feature of 2DPCA has advantage over standard PCA in terms of dealing with high dimensional vector space data. Several other variants of 2DPCA are also applied and the proposed method effectively combines several 2DPCA based techniques [3] [8].

Independent component analysis (ICA) presents a probabilistic image model in which an observed random vector x containing pixels from an image can be decomposed as [4]:

$$X = AS \tag{1}$$

Here s is a vector containing independent sources, which are linearly combined into the observations x through the basis function Ai where the superscript i denotes the i-th column of A. In non-orthogonal paradigm, collection of atoms (or basis functions) is termed as a dictionary which may be incomplete, complete or over complete [4]. The terms basis functions and atoms are used interchangeably. Where A is an $N \times M$ unknown mixing matrix and S is a vector of independent sources.

2. IMAGE COMPRESSION

Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space.

2.1. Image Compression using SVD

SVD takes a matrix, square or non-square, and divides it into two orthogonal matrices and a diagonal matrix. This allows us to rewrite our original matrix as a sum of much simpler rank one matrices [2].

$$A = USV^T \quad (2)$$

2.2. Image Compression using PCA

Let X denotes an n-dimensional unitary column vector called as projection vector [8]. A is $n \times m$ random image matrix which is transformed into Y using X by following linear transformation:

$$Y = AX \quad (3)$$

The average image D, for available L number of images Ak, $k = (1, 2 \ldots L)$ each of size mxn is,

Therefore,

$$D = \frac{1}{L} \sum_{k=1}^{L} Ak \quad (4)$$

Then,

$$G = \frac{1}{L} \sum_{k=1}^{L} \left[(Ak - D)^T (Ak - D) \right] \quad (5)$$

The set of projection axes $X1, X2, \ldots Xd$ are the eigenvectors corresponding the d largest eigen values of G.

The reduced size of feature matrix for an image is the key to image compression. The reconstructed image from its feature matrix can be formed as [8],

$$A = [Y1, Y2, Y3 \ldots Yd]_{mxd} [X1, X2, X3 \ldots Xd]_{dxn} \quad (6)$$

2.3. Image Compression using ICA

In case of independent component analysis the gray scale images of size 92 × 112 is first divided into 4 × 4 block. And then each 4 × 4 block is treated as image now then mean of all the blocks of the images are calculated. The next step is to calculate the variance of all the 4 × 4 blocks. Having the variance values calculate the Eigen value and the Eigen vector. Using these eigen vectors calculate the projection value. Till now the process is same as the PCA. After calculating all the projection values find the maximum projection values with its corresponding index.

3. EXPERIMENTAL VERIFICATION

3.1. Implementation Details

Three set of images of size 112 × 92 of pgm format are considered for the analysis. Validation of the proposed project is done using MATLAB version 7.0 [5]. For each image S matrix is found and graph is plotted for different singular value.

Figure 1: Image of 112 × 92.

Figure 2: Image of 112 × 92.

Figure 3: Image of 112 × 92.

For each image reconstructed matrix is found from U, S and V matrix and reconstructed image is displayed. In all the cases, differences between the original and the reconstructed pixel MSE and PSNR are plotted. In each case, the MSE and PSNR are evaluated for different singular value. In case of PCA set of face images are considered, Mean is calculated in first step and then variance is calculate for this variance matrix eigen vector matrix and eigen value are calculated. The next step is to find feature matrix. Image will be reconstructed using this feature matrix and eigen vector matrix. In case of ICA first image will be divided into 4 × 4 blocks then similarly as in case of PCA, i.e. mean is calculated and then variance of 4 × 4 blocks of images. Find the eigen vector matrix, using this eigen vector matrix and input block image matrix is made row ordered, projection value is calculated for each value in a block of a matrix find the maximum value in it and store it in maximum projection value with its index. Similarly it is repeated for all the blocks of an image.

3.2. Simulation Results

As we saw how these three techniques are applied to an images. let us now compare these technique on the basis of parameter like MSE,PSNR and Compression Ratio (CR).

Let us first define the parameters MSE, PSNR and Compression Ratio.

(1) MSE (Mean Square Error): The MSE is the cumulative squared error between the compressed and the original image.

$$\text{MSE} = \frac{1}{MN}\sum_{x=1}^{N}\sum_{y=1}^{M}[(I(x,y) - I'(x,y)] \tag{7}$$

(2) PSNR (Peak Signal to Noise Ratio): PSNR is a measure of the peak error is given by the formula,

$$\text{PSNR} = 10 \times \log\{(255{*}255)/\text{MSE}\}. \tag{8}$$

(3) Compression Ratio (CR):

$$CR = \frac{\text{total number of bits required to represent an image}}{\text{number of bits required to represent thereconstructed image}}$$

Table 1
Mean Square Error, Peak Signal to Noise Ratio and Compression Ratio of Image (a)

	SVD			
No. of Singular Values	*2*	*4*	*8*	*10*
MSE	401.02	205.35	100.63	78.90
PSNR	22.09	25.00	28.103	29.15
CR	22.06	11.03	5.5	4.4

Table 2
Mean Square Error, Peak Signal to Noise Ratio and Compression Ratio of Image (a)

	PCA			
No. of Eigen Value	*2*	*4*	*8*	*10*
MSE	1212.7	1034.2	893.16	442.91
PSNR	17.29	17.98	21.06	21.667
CR	23	11.5	5.7	4.6

Table 3
Mean Square Error, Peak Signal to Noise Ratio and Compression Ratio of Image (a)

	ICA			
No. of Eigen Value	*1*	*2*	*3*	*4*
MSE	202.76	150.61	118.66	94.71
PSNR	25.06	26.35	27.38	28.33
CR	6.4	3.2	2.13	1.6

5. CONCLUSION

The three lossy techniques Singular Value Decomposition, Principal Component Analysis and Independent Component Analysis give better error resilience, scalability. Singular Value Decomposition and Principal Component Analysis avoids blocking artifacts, from the results of Mean Square Error and Peak Signal to Noise Ratio we can say that Singular Value Decomposition performs better than Principal Component Analysis and Independent Component Analysis and also Independent Component Analysis performs better then Principal Component Analysis. We can also achieve good compression ratio using Singular Value Decomposition when compare to Principal Component Analysis and Independent Component Analysis. Independent Component Analysis achieve better compression ratio when compare to Principal Component Analysis. And we also observed that in three techniques if we increase the number of eigen values decreases the compression ratio and Mean Square Error, increase the Peak Signal to Noise ratio.

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