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An Inventory Model for Constant Deteriorating Items with Price Dependent Demand and Time-varying Holding Cost

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ABSTRACT

In this paper a deterministic inventory model is developed when the deterioration is constant. Demand rate is a function of selling price and holding cost is taken as time dependent. The model is solved allowing shortage in inventory. The results are solved with the help of numerical example by using Mathematica 5.1. The sensitivity analysis of the solution has done with the changes of the values of the parameters associated with the model is discussed.

Keywords: EOQ Model; Deteriorating Items; Inventory Model; Shortage; Price Dependent Demand; Holding Cost.

1. INTRODUCTION

A lot of work has been done for determining the inventory level of deteriorating items which allows and does not allow shortage by different researchers over last three decades Maximum physical goods undergo decay or deterioration over time. Fruits, vegetables and food items suffer from depletion by direct spoilage while stored. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity to use this factor into consideration.

Whitin [1957] considered the deterioration of the fashion goods at the end of a prescribed shortage period. Ghare and Schrader [1963] developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was presented by Shah and Jaiswal [1997], Aggarwal [1978], Dave and patel [1981]. Inventory models with a time dependent rate of deterioration were considered by Covert and Philip [1973], Philip [1974], Mishra [1975] and Deb and chaudhuri [1986]. Some of the recent work in this field has been done by Chung and Ting [1993], Fujiwara [1993], Hariga [1996], Hariga and Benkherouf [1994], Wee [1995], Jalan, et al [1996], Su, et al [1996], Chakraborty and Chaudhuri [1997], Giri and Chaudhuri [1997], Chakraborty, et al [1998] and Jalan and Chaudhuri [1999].

In classical inventory models the demand rate is assumed to be a constant. In reality demand for physical goods may be time dependent, stock dependent and price dependent. Selling price plays an important role in inventory system. Burwell [1997] developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price dependent demand rate was considered by Mondal, et al [2003]. You [2005] developed an inventory model with price and time dependent demand.

In most models, holding cost is known and constant. But holding cost may not always be constant. In generalization of EOQ models, various functions describing holding cost were considered by several researchers like Naddor [1966], Van der Venn [1967], Muhlemann and Valtis Spanopoulos [1980], Weiss [1982] and Goh [1994].

In this present paper, we have developed a generalized EOQ model for deteriorating items where deterioration rate and holding cost are expressed as linearly increasing functions of time and demand rate is a function of selling price. Shortages are allowed here and are completely backlogged.

2. ASSUMPTIONS AND NOTATIONS

The present model is developed under following assumptions and notations:

(i) The demand rate is a function of selling price.

(ii) Shortages are allowed and are fully backlogged.

(iii) The deterioration rate is time proportional.

(iv) Holding cost $h(t)$ per item per time-unit is time dependent and is assumed as $h(t) = h + \alpha t$ where $\alpha > 0, h > 0$.

(v) Replenishment is instantaneous and lead time is zero.

(vi) T is the length of the cycle.

(vii) The order quantity in one cycle is q.

(viii) A is the cost of placing an order.

(ix) The selling price per unit item is p.

(x) C is the unit cost of an item.

(xi) The inventory holding cost per unit per unit time is $h(t)$.

(xii) C_1 is the shortage cost per unit per unit time.

(xiii) $\theta(t) = \beta$ is the constant deterioration, $0 < \beta << 1$.

(xiv) During time t_1, inventory is depleted due to constant deterioration and demand of the item. At time t_1 the inventory becomes zero and shortages start occurring.

(xv) Selling price p follows an increasing trend and demand rate posses the negative derivative through out its domain where demand rate is $f(p) = (a - p) > 0$.

3. MATHEMATICAL FORMULATION AND SOLUTION

Let $Q(t)$ be the inventory level at time t $(0 \le t \le T)$. The deferential equations for the instantaneous state over (0, T) are given by

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -f(p),\ 0 \le t \le t_1 \qquad \ldots(1)$$

$$\frac{dQ(t)}{dt} = -f(p),\ t_1 \le t \le T \qquad \ldots(2)$$

With the condition $Q(t) = 0$ at $t = t_1$.

Using $\theta(t) = \beta$ and $f(p) = (a-p)$, then equation (1) and (2) becomes

$$\frac{dQ(t)}{dt} + \beta Q(t) = -(a-p),\ 0 \le t \le t_1 \qquad \ldots(3)$$

$$\frac{dQ(t)}{dt} = -(a-p),\ t_1 \le t \le T \qquad \ldots(4)$$

With the condition $Q(t) = 0$ at $t = t_1$.

Solving (3) and (4) and neglecting higher powers of β we get

$$\theta(t) = \begin{cases} (a-p)\left[(t_1 - t) + \beta\left(\frac{t_1^2}{2} + \frac{t^2}{2} - tt_1\right) + \beta^2\left(\frac{t_1^3}{6} - \frac{t^3}{6} + \frac{tt_1^2}{2} + \frac{t^2 t_1}{2}\right) \right], & 0 \le t \le t_1 \\ (a-p)(t_1 - t), & t_1 \le t \le T \end{cases}$$

Now stock loss due to deterioration

$$D = (a-p)\int_0^{t_1} e^{\beta t} dt - (a-p)\int_0^{t_1} dt$$

$$= (a-p)\left[\frac{\beta t_1^2}{2} + \frac{\beta^2 t_1^3}{6}\right]$$

$$q = D + \int_0^T (a-p)\,dt$$

$$= (a-p)\left[\frac{\beta t_1^2}{2} + \frac{\beta^2 t_1^3}{6} + T\right] \qquad \ldots(5)$$

(neglecting higher powers of β)

Holding cost is

$$H = \int_0^{t_1} (h + \alpha t) e^{-\beta t} \left\{ \int_t^{t_1} (a-p) e^{\beta u} du \right\} dt$$

$$= h(a-p)\left[\frac{t_1^2}{2} + \frac{\beta t_1^3}{6} + \frac{\beta^2 t_1^4}{24}\right]$$

$$+ \alpha(a-p)\left[\frac{t_1^3}{6} + \frac{\beta t_1^4}{24} + \frac{\beta^2 t_1^5}{120}\right] \qquad \ldots(6)$$

Now shortage during the cycle

Let

$$S = -\int_{t_1}^T (a-p)(t_1 - t)dt$$

$$= \frac{1}{2}(a-p)(T - t_1)^2 \qquad \ldots(7)$$

Total profit per unit time is given by

$$P(T, t_1, p) = p(a-p) - \frac{1}{T}(A + Cq + H + C_1 S)$$

[From (5), (6) and (7)]

$$= p(a-p) - \frac{1}{T}\left[A + C(a-p)\left(\frac{\beta t_1^2}{2} + \frac{\beta^2 t_1^3}{6} + T\right) + \frac{C_1}{2}(a-p)(T-t_1)^2\right.$$

$$\left. + h(a-p)\left(\frac{t_1^2}{2} + \frac{\beta t_1^3}{6} + \frac{\beta^2 t_1^4}{24}\right) + \alpha(a-p)\left(\frac{t_1^3}{6} + \frac{\beta t_1^4}{24} + \frac{\beta^2 t_1^5}{120}\right)\right] \quad \ldots(8)$$

Let $t_1 = \nu T$, $0 < \nu < 1$

Hence we have the profit function

$$P(T, p) = p(a-p) - \frac{1}{T}\left[A + C(a-p)\left(\frac{\beta \nu^2 T^2}{2} + \frac{\beta^2 \nu^3 T^3}{6} + T\right) + \frac{C_1}{2}(a-p)T^2(1-\nu)^2\right.$$

$$\left. + h(a-p)\left(\frac{\nu^2 T^2}{2} + \frac{\beta \nu^3 T^3}{6} + \frac{\beta^2 \nu^4 T^4}{24}\right) + \alpha(a-p)\left(\frac{\nu^3 T^3}{6} + \frac{\beta \nu^4 T^4}{24} + \frac{\beta^2 \nu^5 T^5}{120}\right)\right] \quad \ldots(9)$$

Our objective is to maximize the profit function $P(T, p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial P(T, p)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial P(T, p)}{\partial p} = 0$$

This implies

$$A - \frac{1}{2}h(a-p)\nu^2 T^2 - \frac{1}{3}\alpha(a-p)\nu^3 T^3 - \frac{1}{2}C_1 T^2 (a-p)(1-\nu)^2$$

$$-\beta\left[\frac{1}{2}C(a-p)\nu^2 T^2 + \frac{1}{3}h(a-p)\nu^3 T^3 + \frac{1}{8}\alpha(a-p)\nu^4 T^4\right]$$

$$-\beta^2\left[\frac{1}{3}C(a-p)\nu^3 T^3 + \frac{1}{8}h(a-p)\nu^4 T^4 + \frac{1}{30}\alpha(a-p)\nu^5 T^5\right] = 0 \quad \ldots(10)$$

And

$$a - 2p + C + \frac{1}{2}h\nu^2 T + \frac{1}{6}\alpha\nu^3 T^2 + \frac{1}{2}C_1 T(1-\nu)^2$$

$$+\beta\left[\frac{1}{2}C\nu^2 T + \frac{1}{6}h\nu^3 T^2 + \frac{1}{24}\alpha\nu^4 T^3\right]$$

$$+\beta^2\left[\frac{1}{6}C\nu^3 T^2 + \frac{1}{24}h\nu^4 T^3 + \frac{1}{120}\alpha\nu^5 T^4\right] = 0 \quad \ldots(11)$$

The solutions of (10) and (11) will give T^* and p^*. The values T^* and p^*, so obtained, the optimal value $p^*(T, p)$ of the average net profit is determined by (9) provided they satisfy the sufficient conditions for maximizing $P(T, p)$ are

$$\frac{\partial^2 P(T, p)}{\partial T^2} < 0, \quad \frac{\partial^2 P(T, p)}{\partial p^2} < 0 \quad \ldots(12)$$

And

$$\frac{\partial^2 P(T, p)}{\partial T^2}\frac{\partial^2 P(T, p)}{\partial p^2} - \left(\frac{\partial^2 P(T, p)}{\partial T\, \partial p}\right)^2 > 0 \text{ at } p = p^*$$

and $T = T^*$ …(13)

If the solutions obtained from equation (10) and (11) do not satisfy the sufficient conditions (12) and (13) we may conclude that no feasible solution will be optional for the set of parameter values taken to solve equations (10) and (11). Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

Numerical Example: Suppose $A = 250$, a=100, $C = 20$, $\beta = 0.95$, $\alpha = 0.1$, $c_1 = 1.3$, $h = 0.4$, ν =0.01 the in appropriate units. Based on these input data, the computer outputs are as follows: Profit =769.683, $T = 1.211$ and $p = 0.972$

Table

Changing Parameter	*% change in system*	*p*	*q*	*T*	*Profit*
	+50	0.806505	1.02235	1.12615	730.362
α	+20	0.822407	1.04391	1.15044	740.91
	–20	0.852026	1.08407	1.14675	756.508
	–50	0.887656	1.13238	1.25007	769.883
	+50	0.972665	1.14654	1.21172	773.665
β	+20	0.903648	1.10686	1.19339	761.443
	–20	0.737897	0.986969	1.12967	728.983
	–50	0.4852	0.740255	0.977145	683.63
	+50	0.940846	1.16887	1.21535	743.636

Contd...

Contd...

ν	+20	0.877751	1.10471	1.21395	746.521
	–20	0.793381	1.01879	1.12719	750.397
	–50	0.729862	0.954018	1.06182	753.332
	+50	0.959257	1.03975	1.13075	438.424
h	+20	0.934248	1.05977	1.1519	584.611
	–20	0.403518	0.224739	2.4374	1584.69
	–50	0.413311	0.19967	2.31293	2820.44
	+50	0.813115	1.03131	1.17035	1123.27
c_1	+20	0.824126	1.04624	1.17046	898.408
	–20	0.853833	1.08652	1.17086	598.384
	–50	0.918366	1.17402	1.17262	372.194
	+50	0.877574	1.11871	1.17131	747.314
c	+20	0.851576	1.08346	1.17082	748.109
	–20	0.820549	1.04139	1.17042	748.699
	–50	0.799448	1.01278	1.17025	748.948
	+50	0.863967	1.10026	1.2139	1116.71
A	+20	0.848206	1.07889	1.18983	896.038
	–20	0.820512	1.04134	1.14755	600.418
	–50	0.789885	0.999813	1.10078	377.309

4. SENSITIVITY ANALYSIS

To study the effects of changes in the system parameters, c, h, c_1, α, β, A and ν on the optimal cost derived by the proposed method and a is constant. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values.

On the basis of the results of table, the following observation can be made.

(i) Decrease in the value of the parameters α then p, q, T and $P(T, p)$ is increased.

(ii) Decrease in the values of either of the parameters β, A then p, q, T is decreased.

(iii) Decrease in the value of the parameters c_1 then p, q, T is increased and $P(T, p)$ is decreased.

(iv) Decrease in the value of the parameters ν then p, q, T is decreased and $P(T, p)$ is increased.

(v) Decrease in the value of the parameters h then p, q is decreased and T, $P(T, p)$ is increased.

(vi) Decrease in the value of the parameter C then p, q, T is decreased and $P(T, p)$ is increased ..

5. CONCLUSION

In the present paper a deterministic inventory model is developed for deteriorating items. Shortages are allowed and are completely backlogged in the present model. In many practical situations, stock out is unavoidable due to various uncertainties. there are many situations in which the profit of the stored item is high than its backorder cost Which is most applicable to the physical goods under delay or deterioration over time. Commodities such as fruits, vegetables, food stuff, etc. Suffer from depletion by direct spoilage while kept in store.

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