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FRACTIONAL TIME MINIMIZING TRANSPORTATION PROBLEM

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Abstract: Fractional Time Minimizing Transportation Problem (FTMTP) is presented in this paper. FTMTP is related to lexicographic fractional time minimizing transportation problem, which will be solved by a lexicographic primal code. An algorithm is also developed to determine an initial efficient basic solution of this FTMTP. Numerical example has been provided to illustrate the solution procedure.

Keywords: Time Transportation, Lexicographic, Optimization, Fractional Programming.

1. INTRODUCTION

Time minimizing transportation problem is required to find a feasible transportation schedule which minimizes the maximum of transportation time associated between a supply point and a demand point such that the distribution between the two points is positive. Seshan and Tikekar [3] presented a time minimizing transportation problem to determine the set S_{hk} of all non basic cells. Achary and Seshan [1] discussed a time minimizing transportation problem with a more general and realistic assumption that the time $t_{ij}(x_{ij})$ required for transporting x_{ij} units depends on the actual amount transported. Sonia and Puri [5] considered a two level hierarchical balanced time minimizing transportation problem.

Transportation problems with fractional objective function are widely used as performance measures in many real life situations where an individual, or a group of community is faced with the problem of maintaining good ratios between some very important crucial parameters concerned with the transportation of commodities from certain sources to various destinations. Corban [2] extended the concept of multi-dimensional transportation problem with fractional linear objective function and derived the optimality conditions. Sharma and Swarup [4] presented a transportation technique for time minimization in fractional functional programming problem with an objective function.

This paper deals with a Fractional Time Minimizing Transportation Problem. FTMTP is related to a lexicographic fractional time minimizing transportation problem which will be solved by a primal algorithm. By means of primal algorithm, the vector of partial flows is minimized in a lexicographic sense on the feasible set.

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2. MATHEMATICAL FORMULATION

Given a actual transportation time matrix T^a and a standard transportation time matrix T^s, where $T^a = [t_{ij}^a]$ and $T^s = [t_{ij}^s]$, for transporting the goods from ith supply point (i = 1, 2, ..., M) to jth demand point (j = 1, 2, ..., N), the problem is to find a feasible distribution which minimizes the maximum fractional transportation time. The mathematical formulation of FTMTP is:

$$\min t = \max_{(i,\,j)} \left\{ \frac{t_{ij}^a}{t_{ij}^s} \middle| \, x_{ij} > 0 \right\} \tag{1}$$

subject to

$$\sum_{i=1}^{M} x_{ij} = b_i \qquad (i = 1, 2, ..., N)$$

$$\sum_{i=1}^{M} x_{ij} = b_i \qquad (i = 1, 2, ..., N)$$

$$x_{ij} \geq 0 \quad (i = 1, 2....., M;\ j = 1, 2, ..., N)$$

where

a_i = amount of the commodity available at the *ith* supply point

b_j = requirement of the commodity at the jth demand point

x_{ij} = amount of commodity transported from the ith supply point to the jth demand point

$t_{ij}^a =$ actual transportation time from the ith supply point to the jth demand point

$t_{ij}^s =$ standard transportation time from the ith supply point to the jth demand point

$\frac{t_{ij}^a}{t_{ij}^s} =$ proportional contribution to the value of the fractional time objective function for shipping one unit of commodity from the ith supply point to the jth demand point

t = fractional bottleneck transportation time.

Assumption: a_i and b_j are given non-negative numbers and are not simultaneously zero and total demand requirement equals the total supply.

Setting $M' = \{1, 2, ..., M\}$, $N' = \{1, 2, ..., N\}$, $J' = \{(i, j) \mid i \in M', j \in N'\}$ the above FTMTP may be related to the following lexicographic fractional time minimizing transportation problem and $\alpha_{ij} \in IR^h$, $\beta_{ij} \in IR^h$:

$$\text{lexmin}\left[\Im = \frac{\sum_{(i,j)\in J'} \alpha_{ij} x_{ij}}{\sum_{(i,j)\in J'} \beta_{ij} x_{ij}} \left| \begin{array}{l} \sum_{j\in N'} x_{ij} = a_i \text{ for all } i \in M' \\ \sum_{i\in M'} x_{ij} = b_j \text{ for all } j \in N' \\ x_{ij} \geq 0 \quad \text{for all } (i, j) \in J' \end{array} \right. \right] \quad (2)$$

Remark: Let IR denote the set of the real numbers and IR^0 the set of the non-negative real numbers. With regard to lexicographic vector inequalities, the following convention will be applied: For $a, b \in IR^h$ the strict lexicographic inequality $a \underset{\sim}{>} b$ holds if and only if $a_{\tilde{c}} > b_{\tilde{c}}$ holds for $\tilde{c} = \min\{c \mid c = 1, 2, ..., h, a_c \neq b_c\}$ and the weak lexicographic inequality $a \underset{\sim}{\geq} b$ holds if and only if $a \underset{\sim}{>} b$ or $a = b$.

3. FTMTP ALGORITHM

The optimal basic solution to (2) provides a feasible transportation schedule that minimizes the fractional transportation time as well as the total distribution that requires the fractional transportation time. The steps of the algorithm are:

Step 1: Determine the lower bound t_l^a on t^a and t_l^s on t^s to reduce the dimension of the vectors α_{ij} and β_{ij} respectively in (2). Now $t_l^a > t_{ij}^a$ and $t_l^s > t_{ij}^s$ for at least one pair $(i, j) \in \xi^a, \xi^s$ respectively. Here ξ_g^a consists of all $(i, j) \in \xi^a$ with $t_l^a > t_{ij}^a$ and ξ_h^s consists of all $(i, j) \in \xi^s$ with $t_l^s > t_{ij}^s$ respectively.

Step 2: Determine an initial basic feasible solution to (2) by North-West Corner Rule.

Step 3: Determine an upper bound t_U^a and t_U^s, from the resulting transportation time t^a and t^s respectively of the initial basic feasible solution. Let $t_U^a < t_{ij}^a$ and $t_U^s < t_{ij}^s$ for at least one pair $(i, j) \in \xi^a, \xi^s$ respectively. Then ξ_1^a consists of all $(i, j) \in \xi^a$ with $t_U^a < t_{ij}^a$ and ξ_1^s consists of all $(i, j) \in \xi^s$ with $t_U^s < t_{ij}^s$ respectively.

Step 4: Partition the set $\xi^a = M \times N$ and $\xi^s = M \times N$ into subset ξ_c^a and ξ_d^s respectively ($c = 1, ... g; d = g + 1, ..., h$) and determine the vectors α_{ij} and $\beta_{ij}, (i, j) \in \xi_c$, and ξ_d respectively such that: $\alpha_{ij} = [e_c]$ and $\beta_{ij} := [e_d]$ to obtain fractional bottleneck transportation time matrix T.

Step 5: Designate the set of pairs of indices (i, j) of the basic variable by I and using initial basic feasible solution, determine recursively the vector-valued row and column multipliers $u_i^1, u_i^2, v_j^1, v_j^2$ defined such that:

$$[\alpha_{ij} - (u_i^1 + v_j^1)] = 0 \quad (3)$$

$$[\beta_{ij} - (u_i^2 + v_j^2)] = 0 \quad (4)$$

for those i, j for which x_{ij} is in the basis.

Step 6: Let $\tilde{U} = (\tilde{u}_i^1, \tilde{u}_i^2, i \in M'; \tilde{v}_j^1, \tilde{v}_j^2, j \in N')$ be the solution of (3) and (4). Compute the relative criterion vectors Δ_{ij} by using the following equation set:

$$\Delta_{ij} = [V_2\alpha'_{ij} - V_1\beta'_{ij}] \quad (5)$$

where

$$\alpha'_{ij} = [\alpha_{ij} - (\tilde{u}_i^1 + \tilde{v}_j^1)] \quad (6)$$

$$\beta'_{ij} = [\beta_{ij} - (\tilde{u}_i^2 + \tilde{v}_j^2)] \quad (7)$$

$$V_1 = \sum_{i\in M'} \tilde{u}_i^1 a_i + \sum_{j\in N'} \tilde{v}_j^1 b_j \quad (8)$$

$$V_2 = \sum_{i\in M'} \tilde{u}_i^2 a_i + \sum_{j\in N'} \tilde{v}_j^2 b_j \quad (9)$$

for all $(i, j) \in J' \setminus I$

Step 7: If all Δ_{ij} are lexicographically greater than or equal to the zero vector for all $(i, j) \in J' \setminus I$, the current basic feasible solution is optimal to (2) and go to Step 9. Otherwise go to Step 8.

Step 8: Select

$$\Delta_{i^*j^*} = \text{lexmin}\left\{\Delta_{ij} \middle| \Delta_{ij} \underset{\sim}{<} 0\right\} \quad (10)$$

for all $(i, j) \in J' \setminus I$ and determine the variable $x_{i^*j^*}$ which is to be enter. Now the variable $x_{i^*j^*}$ becomes a basic variable of the new basic feasible solution. Change the current basic feasible solution to the new basic feasible solution using the standard transportation method and go to Step 5.

Step 9: If $\tilde{X} = (\tilde{x}_{ij})$ is optimal transportation schedule for (2), then $\tilde{\Im} = \sum_{(i,j)\in J'} \alpha_{ij}\tilde{x}_{ij} \Big/ \sum_{(i,j)\in J'} \beta_{ij}\tilde{x}_{ij}$ and $\tilde{c} / \tilde{d}$ is the index of the first positive component of the optimal flow vector $\tilde{\Im}$. Also $\tilde{t} = \frac{\tilde{t}_{ij}^a}{\tilde{t}_{ij}^s}$ with

$(i, j) \in \xi_{\tilde{c}}^{a} / \xi_{\tilde{d}}^{s}$ is the optimal fractional bottleneck transportation time.

4. NUMERICAL EXAMPLE

For an illustration of the above algorithm, solve the following problem:

$$\min t = \max_{(i,j)} \left\{ \frac{t_{ij}^{a}}{t_{ij}^{s}} \mid x_{ij} > 0 \right\}$$

subject to $\sum_{j=1}^{4} x_{ij} = a_i \quad (i = 1, 2, ..., 4)$

$$\sum_{i=1}^{4} x_{ij} = b_j \quad (j = 1, 2, ..., 4)$$

$$x_{ij} \geq 0 \quad (i = 1, 2, ..., 4; j = 1, 2, ..., 4)$$

Table 1 shows the fractional time (in hours) from i to j. Availabilities a_i are shown in the last column while requirements b_j are shown in the last row.

Table 1
Fractional Time (in hours)

Source i	*Destination j*				
	1	*2*	*3*	*4*	a_i
1	$\left[\frac{4:40}{3:20}\right]$	$\left[\frac{4:50}{3:30}\right]$	$\left[\frac{4:20}{3:40}\right]$	$\left[\frac{4:45}{3:50}\right]$	7
2	$\left[\frac{4:55}{3:30}\right]$	$\left[\frac{4:35}{3:35}\right]$	$\left[\frac{4:45}{3:50}\right]$	$\left[\frac{4:00}{3:00}\right]$	1
3	$\left[\frac{5:00}{3:30}\right]$	$\left[\frac{4:45}{3:40}\right]$	$\left[\frac{4:30}{3:20}\right]$	$\left[\frac{4:50}{3:40}\right]$	8
4	$\left[\frac{4:40}{3:45}\right]$	$\left[\frac{4:50}{3:20}\right]$	$\left[\frac{4:20}{3:10}\right]$	$\left[\frac{4:45}{3:35}\right]$	4
b_j	5	6	3	6	

The lower bound for actual time matrix T^a is obtained by calculating row threshold (4:40, 4.00, 4:45, 4:40) and column thresholds (4:40, 4:45, 4:20, 4:45) which gives $t_l^a = 4:45$.

Similarly for standard time matrix T^s, the lower bound is $t_l^s = 3:35$. The initial basic feasible solution X^1 is: $x_{11} = 5, x_{12} = 2, x_{22} = 1, x_{32} = 3, x_{33} = 3, x_{34} = 4$ with the resulting actual transportation time 4:50, which gives the upper bounds $t_U^a = 4:50$ and resulting standard transportation time 3:40, which gives the upper bounds $t_U^s = 3:40$.

Using initial basic feasible solution X^1, the associated vector-valued row multipliers u_i^1, u_i^2, and associated vector-valued column multipliers v_j^1, v_j^2, are calculated as explained in Step 5.

Once the vector-valued row and column multipliers are determined, the values of relative criterion vectors Δ_{ij} are obtained from the equation (5). As X^1 is not optimal, therefore using the standard transportation method, variable x_{14} becomes an entering basic variable. Change the current basic feasible solution to the new basic feasible solution using the standard transportation method. Proceeding in the same manner the subsequent iterations are as follows:

$$X^2 : x_{11} = 5, x_{14} = 2, x_{22} = 1, x_{24} = 0, x_{32} = 5, x_{33} = 3, x_{44} = 4.$$

Flow vector $\Im(X^2) = (0,0,0,0,0,8,4,0,0,1,5,0,0,2,0)^T$

$$X^3 : x_{11} = 1, x_{14} = 6, x_{22} = 1, x_{24} = 0, x_{32} = 5, x_{33} = 3, x_{41} = 4.$$

Flow vector $\Im(\tilde{X}^3) = (0,0,0,0,0,4,0,0,0,1,5,0,0,6,4)^T$

The current basic feasible solution X^3 is optimal. The optimal value of the flow vector is $\Im(\tilde{X}^3)$ = (0, 0, 0, 0, 0,0 , 4 , 0, 0, 1, 5, 0, 0, 6, 4)^T. Thus the optimal fractional transportation time is $\tilde{t} = 1.334$ and the optimal flow = 4.

5. CONCLUSION

The algorithm helps in determining all efficient transportation schedules with respect to the minimization of non-linear time function and the distribution that requires the fractional time. The algorithm offers a more universal apparatus for a wider class of real life decision priority problems.

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